Problem Set 10

1. What is the largest set of natural numbers from 1 to 202 inclusive, so that the sum of each pair of them is divisible by 6?

2. Decode the equalities (DA)(BC) = (AC)(AEE) = ABBC if each letter represents a different digit and the same letter represents the same digit.

3. Find all natural numbers N so that the number $2^4 \text{ X } 3^{16} + 5^2 \text{ X } 3^{14} + 3^N$ is a perfect square.

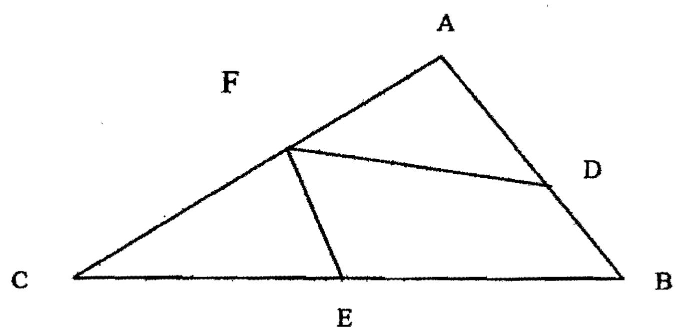
4. If two five-digit numbers, A and B, are formed using the digits 0 through 9 each once, and given that $\frac{A}{B} = \frac{1}{2}$, what is the largest possible sum of A + B?

5. The speed of A is 13 km/h and the speed of B is 11 km/h. If B has run 20 minutes longer than A and, as a result, has traveled 2 km farther than A, how far has B run?

6. In an exam A, B, C, D and E all scored more than 90 out of a possible 100 points. All their scores were whole numbers. The average for A, B and C was 97, the average for B, C and D was 94, A had the highest score and E scored 96 which was also the median. What was D's score?

7. There are 11 liters of alcohol in container A and 15 liters of water in container B. Some of the alcohol is poured into B and mixed. Then some of this mixture is poured into A The contents of A are now 62.5% alcohol and the contents of B are 25% alcohol. How many liters of the mixture were poured into A?

8. In triangle ABC, $\angle B = 76^0$, AD = AF and CE = CF. What is the measure of $\angle$DFE?



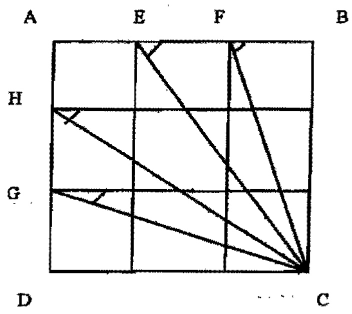
9. What is the probability that rolling two dice does not produce a sum of 7 until the fourth roll?

10. Find a value of x for which $\sqrt{x+47}$ and $\sqrt{x-12}$ are both positive integers.

11. Regular hexagon ABCDEF is inscribed in rectangle WXYZ so that A and B are points on XY, C is on YZ, D and E are on ZW and F is on WX. Find the ration of the area of the hexagon to the area of the rectangle.

12. A square pyramid is cut by a plane which is parallel to its base and 2 units from it. The surface area of the smaller pyramid that results is 1/2 the surface area of the original pyramid. What was the height of the original pyramid?

13. The diagram shows square ABCD formed by 9 identical smaller squares. Vertices E, F, G, and H are each joined to vertex C. What is the sum of the four marked angles?



14. Points P and Q lie on sides AB and BC respectively of ΔABC. If AQ and CP intersect at R, AP:PB = 3:4 and BQ:QC = 1:2, find the ratio of the area of quadrilateral BQRP to the area of ΔABC.

15. Yan's home is on a street that leads to the soccer stadium. He is presently somewhere between his home and the stadium and he can either walk directly to the stadium or walk home and ride his bicycle to the stadium. If he rides 7 times as fast as he walks both choices require the same amount of time to reach the stadium. Find the ratio of Yan's distance from his home to his distance from the stadium.

16. The diagonals of square ABCD intersect at O and A is the midpoint of side PQ of equilateral triangle OPQ. Find the ratio of the area of the OPQ that lies inside ABCD to the area of OPQ that lies outside ABCD.

17. During the first minute Mark places 2 red dots on a sheet of paper. During the second minute he places a green dot midway between them. During the third minute he places 2 red dots in the spaces between the previous three dots, one in each space. During the fourth minute he places 4 green dots in the spaces between the previous dots, again one in each space. If he continues to alternate placing red and green dots in this fashion, how many green dots will there be at the end of one hour?

Alternate question: How many green dots are added during the 60^{th} minute?

18. On a floor consisting of black, grey and white tiles a bug hops about, always landing on a tile of a different color from the tile it left. The probability that it hops from black to gray is 1/3, the probability that it hops from grey to white is 1/4, and the probability that it hops from white to black is 6/7. If it begins on a black tile what is the probability that it will again be on a black tile after three hops? After four hops?

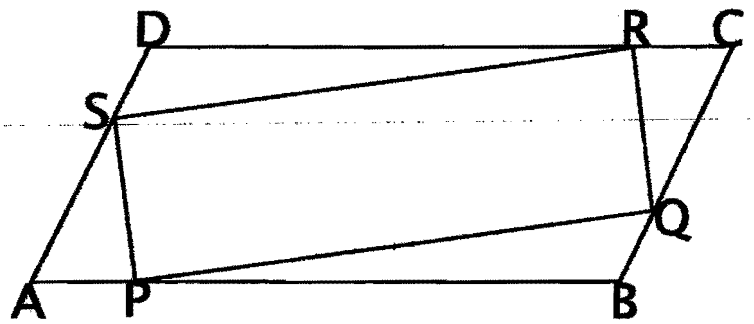
Problem Set 11

1. A biased coin is flipped. The probability of getting a head (H) is p. Player A wins if the sequence HHH occurs, player B wins if the sequence HTH occurs. Find the value of p if A and B have an equal chance of winning.

2. A sequence of 21 numbers has the following properties: The first term is zero. The sequence is unchanged if we reverse the order of the terms. Each term (other than the first and last) is one more than the average of the two adjacent terms. Find the 11th term in the sequence.

3. Let a, b, and c be two digit numbers and the addition a + b + c = 75 requires no carrying in base 10. How many ordered triplets (a, b, c) are possible?

4. Points P, Q, R and S lie on sides AB, BC, CD and DA respectively of parallelogram ABCD so that AP/PB=BQ/QC=CR/RD=DS/SA=1/2. What is the ratio of the area of PQRS to the area of ABCD?

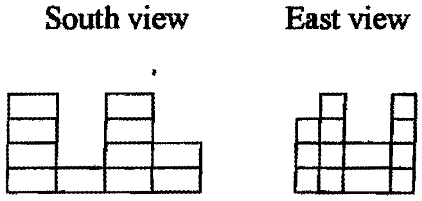


5. Ten points are chosen on a circle. The chord between each pair of points is drawn. How many points inside the circle are intersection points of two chords if no such point lies on three or more chords?

6. All the integers are colored white, except 1, which is colored red. Any integer which exceeds a red number by 20 or by 21 will be repainted red. What is the largest number which will remain white?

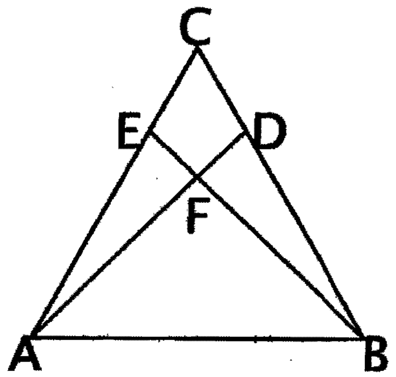
7. For how many different subsets of (2, 3, 4, 6, 15, 20, 30) do the elements total at least 50?

8. Some unit cubes are stacked on a flat 4 by 4 square. The figures show views of the stacks from two different sides. Find the maximum and minimum number of cubes that could be in the stacks. (Clearly not a scale drawing)



9. Find a four digit number abcd, written in base eight, so that placing a decimal point between b and c produces a number that is the average of the two digit numbers ab and cd, also written in base eight.

10. Equilateral triangle ABC has sides of 3 and D and E are points on BC and AC respectively so that CD=CE=1. If AD and BE intersect at F, what is the area of the quadrilateral CDFE?



11. In triangle ABC side AB = 25 while altitude BE=24 and altitude AD = 20. Find the perimeter of triangle ABC.

12. For how many subsets of (2, 3, 4, 6, 15, 20, 30) do the elements total at least 50?

13. What is the largest number between 1 and 100 with exactly 12 positive divisors, (including 1 and itself).

14. A store will give a surprise gift with any purchase totaling $25 or more. The store has 6 items you like, with prices of $2, $3, $7, $9, $11 and $24. How many selections can you make from items you like to qualify for the gift, if you will not purchase two or more of the same item?

15. Players A and B are tied 20 – 20 in a ping-pong match. The first player to lead by 2 points will win. How many scoring sequences after the 20 – 20 tie will lead to A winning 30 – 28?

16. Points D and E lie on sides AB and BC respectively of triangle ABC. If AD = 2BD, CE = 3EB, and F is the midpoint of CD, what is the ratio of the area of ADEF to the area of ABC?

17. Nine identical balls with a radius of 1 unit are packed in a box that has a base that is 4 units by 4 units and a height of h units. What is the minimum vale of h so the top of the box can be closed?

18. At a carnival booth you are allowed to throw a ball 3 times at a target. You must hit the target twice in a row to win. You must also alternate between throwing with your right and left hand. You can hit the target 6/10 of the time throwing with your right hand and 3/10 of the time throwing with your left hand. You may start with either hand. What is the probability that you win if you make the best choice of starting hand?

19. How many 5-digit integers have at least two 7's appearing consecutively in their usual decimal representation?

Problem Set 12

1. Pentagon ABCDE has AB = 16 units and BC = EA = 10units. If EC is parallel to AB with EC = 28 units and AD = BD = 17 units, what is the area of the pentagon?

2. In how many ways can 4 concentric disks of different diameters be stacked in 3 piles; A, B, and C, if no disk may be placed on top of a smaller disk. It is not required that every pile contain a disk.

3. Consider the set of all four-digit numbers in which the digits are x, x + 2, x + 4, and x + 6. What is the probability that a number in the set is a multiple of 5?

4. How many four-digit numbers can be written if each number uses exactly two different digits?

5. Team A is favored by odds of 3 to 2 when it plays Team B. In a series of games between the two teams, what is the probability that Team B is the first to win three games?

6. In the multiplication below each letter and each □ represents a single digit. Different letters represent different digits but a □ can represent any digit. What number does HAPPY represent?

```
    □ 1 □
      9 □
  -------
  □ □ 9 □
□ □ □ 7
---------
H A P P Y
```

7. A truck travels from point A to point B at an average speed of 50 mph and returns from B to A at an average speed of 70 mph. The truck makes 3 round trips in 18 hours. What is the distance , in miles, between A and B?

8. John and Mary went to a bookstore and bought some exercise books. They had $100 each. John could buy 7 large and 4 small ones. Mary could buy 5 large books and 6 small ones and have $5 left. What is the cost of a small exercise book?

9. A chemist mixed an acid of 48% concentration with the same acid of 80% concentration, then added 2 liters of distilled water to the mixed acid. As a result he now has 10 liters of the acid of 40% concentration. How many milliliters of the 48% acid solution did he use?

10. In a class 40% of the girls and 50% of the boys made an "A" on a test. If a total of 12 students in the class made an "A" and the ratio of girls to boys in the class is 5:4, how many students are in the class?

11. A rectangular region is covered by square tiles, each 1 ft by 1 ft. The region is two feet longer than it is wide. Exactly half of the tiles meet the perimeter of the region. What is the area of the region?

12. What is the sum of 1/2! + 2/3! + 3/4! + ... + 7/8! ?

13. How many scoring sequences are possible if the Hawks won their soccer game by a score of 5 to 4 and were never behind in the game?

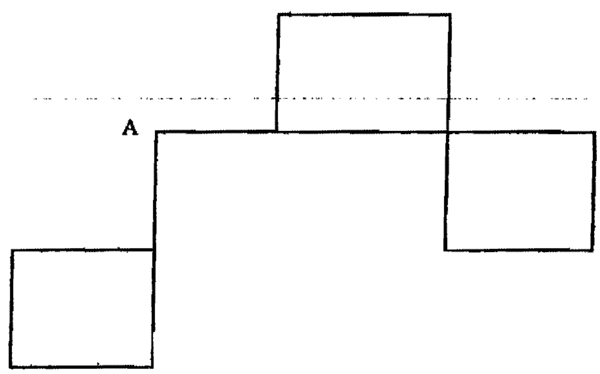
14. In rectangle ABCD points E and F lie on BC so that CE:EF:FB = 1:2:3. Diagonal BD intersects AF at H and AE at G. If the area of ABCD is 132 sq units, what is the area of EGHF?

15. In the multiplication shown below each letter represents a single digit. What is the sum A + B + C + D?

```
  A4B
   36
 ----
15CD4
```

16. A bottle is filled with water. One-fourth of this is removed and replaced with alcohol. After mixing, one-fourth of the mixture is removed and the bottle is again refilled by adding alcohol. If this process is carried out four times, what fraction of the final mixture in the bottle is alcohol?

17. Alvin starts at point A and walks along the pictured paths. Each time he comes to an intersection, he randomly decides which of the available paths he will follow. He never backtracks over the path he just took, but may retrace a path later in his journey. After he has traveled along six segments, what is the probability that he is back at A?



18. There are 13 checkers arranged around a circle. If three are red and ten are black and no two red checkers are together, how many distinct ways can the checkers be arranged around the circle?

19. Twelve points are selected on a circle. Sam is required to draw two triangles, where the vertices of both triangles must be chosen from the twelve points, but the triangles are not allowed to intersect. In how many ways can this be done?

20. What is the sum 1 + 1/3 + 1/6 + 1/10 + ... + 1/300 where the denominator of the nth term is the sum of the first n positive integers?

Problem Set 13

1. The age of a father is five years less than the sum of the ages of his wife and daughter. In seven years the wife will be three times as old as the daughter and the sum of the ages of all three will be 108. How old is each now?

2. A ticket for a football match originally cost $15. After a reduction in price, ticket sales increased by 50% and the income by 20%. What was the reduced price?

3. There is a 31 digit number in which any two adjacent digits form a multiple of either 17 or 23. The digit "7" appears only once in this number. What is the sum of the 31 digits of this number?

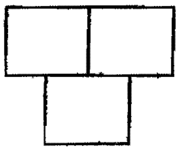
4. One year in the Primary Math World Contest the sum of all the scores was 8640. There were only three people who scored at least 80 points and their scores were 92, 85 and 81. The lowest score was 25 and no score occurred more than three times. At least how many contestants scored 60 or more points?

5. Siu-ming has bought a red pen and a blue pen. They total cost was $17. The unit price of each pen is a whole number of dollars. He would like to some more of the pens for $35 but can not find any combination of red and blue pens that would cost that amount. What does a red pen cost if it is more expensive than the blue pen?

6. What is n if $\frac{1+2+3+....+n}{3n} = 36$

7. Mary took 24 chickens to the market. In the morning she sold the chickens for $7 each and she was able to sell less than half of them. In the afternoon she reduced the price per chicken but it was still an integral number of dollars. By the end of the day she had sold all the chickens for a total of $132. How many chickens were sold in the morning?

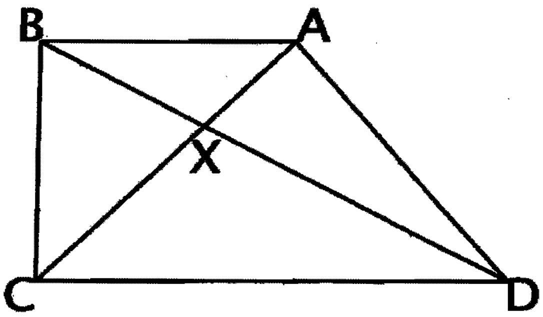
8. The given figure consists of three identical squares with sides of 1 unit. There are 10 points where two or more segments intersect. How many triangles with an area of one square unit can be formed by selecting 3 of the 10 points as vertices if one side of the triangle must be either a horizontal or vertical segment?



9. Yang has a pair of four-sided dice, (each face is an equilateral triangle), and the faces of each die are numbered 1 through 4. Zijie has a six-sided die with the faces numbered 1 through 6. If all the dice are rolled what is the probability that the total of the numbers on the six visible faces of Yang's dice is at least as great as the total of the numbers on the five visible faces of Zijie's die?

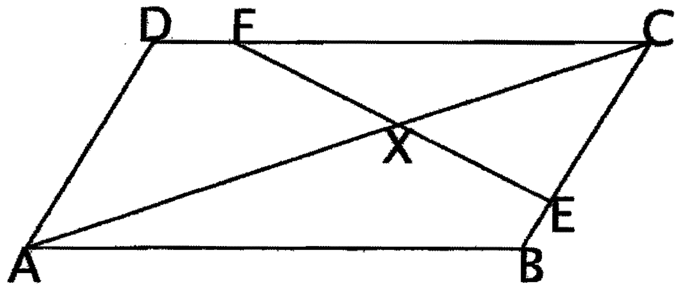
10. A survey of a 100 students about their favorite soft drink shows that 50 prefer drink A, 40 prefer drink B and 10 prefer drink C. If 4 students are chosen at random from the group what is the probability that exactly 2 of them prefer drink A? Write you answer as a fraction reduced to lowest terms.

11. In trapezoid ABCD segments AB and CD are both perpendicular to BC. Diagonals AC and BD intersect at X. If AB=9, BC=12 and CD=16, what is the area of triangle BXC?



12. Triangle ABC has AB = 8, BC = 15 and $\angle B = 90^0$. A square XYZW is inscribed with X and Y on AC, Z on AB and W on BC. Find the length of a side of the square.

13. Points E and F lie on sides BC and CD respectively of parallelogram ABCD and EF intersects AC at X. IF CF = 4DF and CE = 3BE, find the value of CX/AX in lowest terms.



14. Twelve colors are available for painting the faces of a regular dodecahedron. In how many ways can the dodecahedron be painted if each face is painted a single color and no two faces are painted the same color? We consider that two dodecahedra have been colored in the same way if we can perform a rotation so that the colors of corresponding faces match.

15. The snails Slow and Slower left point A for point B at 7 a.m. Slow was crawling at a constant speed of 12 m/h. Slower had started at a constant speed of 8 m/h but two hours later he climbed onto the back of the turtle Tom moving in the same direction at a speed of 20 m/h. Slower and Tom passed Slow and four hours later arrived at B. Slower crawled down from Tom's back and started crawling back toward A at a speed of less than 4 m/h. What is the distance from A to B? Between which two hours did Slower meet Slow on his return toward A?

Problem Set 14

1. Two cars left city A for B at 8 a.m. The distance between the cities is more than 200 km. The speed of one of the cars was 60 km/h during the first two hours and 15 km/hour for the rest of the trip to B. The speed of the other car was 15 km/h during the first two hours and 60 km/h during the rest of the trip to B. At what time/s was the distance between the two cars 15 km?

2. A grandmother has two grandsons. Her age is a two-digit number. The first digit is equal to the age of one grandson and the second digit to the age of the other grandson. How old are each of the three if the sum of their ages is 69?

3. A cubical die has 1 and 6, 2 and 5, and 3 and 4 on opposite faces. When two dice are rolled the top number of the first is multiplied by both the top and bottom number of the second and the bottom number of the first is also multiplied by the top and bottom number of the second. These four products are added. What is the sum?

4. Subtract the same number from the numerator and denominator of $\frac{37}{76}$. When you reduce this fraction to lowest terms it equals $\frac{2}{3}$. What was the number you subtracted?

5. ~~Find the sum $1+2-3+4+5-6+\ldots+(3n-2)+(3n-1)-3n+\ldots+97+98-99$.~~

6. Let a "lucky" number be an integer with the sum of its digits being exactly divisible by 7. What is the maximum number of "lucky" numbers that could be found in a sequence of ten consecutive 3-digit numbers?

7. A word is just a string of letters. So we would call "WDOO" a word even though it does not have any real meaning. How many 6-letter words can you make using the letters of "contest" with no letter appearing in a word more often than it appears in contest?

8. The area of rectangle ABCD is 36 sq units. If X is the midpoint of BC and AX intersects BD at O, what is the area of the quadrilateral CDOX?

9. Peter is ill. He has to take medicine A every 8 hours, medicine B every 5 hours and medicine C every 10 hours. If he took all three medicines at 7 a.m. on Tuesday, when is the first time he will be taking all three together again?

10. There are two times between 10 p.m. and midnight when the hour and minute hands of a 12 hour clock form the same angle but the position of the hands is reversed. What is the earliest time that this occurs?

11. Suppose $(2-3x^2)^{2003} = C_0 + C_1x + C_2x^2 + \ldots + C_{4006}x^{4006}$ where C_i is the coefficient of x^i in the expansion of $(2-3x^2)^{2003}$. What is the value of $C_0 + C_1 + C_2 + \ldots + C_{4006}$?

12. Half of a class watched a soccer match on television this weekend. Only 3/8 of the girls watched but 3/5 of the boys watched the match. What fraction of the class is girls?

13. An isosceles trapezoid has bases of 12 and 18 units and a diagonal of 17 units. What is the area of the trapezoid?

14. A sequence begins 1, 2, 3. The remaining terms in the sequence will be found by subtracting the last known term from the sum of the two that precede it. Thus this sequence continues 1, 2, 3, 0, 5, ... What is the 2002^{nd} term in the sequence?

15. A grocer mixes two kinds of tea. One is 32¢ per pound tea and the other is 40¢ per pound tea. He sells this mix for 43¢ per pound, making a profit of 25% of the cost. How many pounds of each kind of tea did he use to make a 100 pound mix?

16. In triangle ABC, AB = AC. Points D and E lie on AC, (with D between A and E), and F lies on AB, so that AD = DF = FE = EB = BC. What is the measure of angle BAC?

17. In triangle ABC, AB = AC = 1. Point D is on AC so that AD = DB = BC = x. What is the value of x?

18. After multiplying the product of two three-digit numbers by 3, a six-digit number is obtained such that the first three digits form a number equal to one of the three digit factors and the last three digits form a number equal to the other three-digit factor. What are these two numbers?

19. Find all five-digit numbers, whose digits do not include zeros, that are perfect squares and remain perfect squares when their first digit is removed, when their first and second digits are removed and when their first three digits are removed.

20. A mouse is 20 steps from its hole and a cat is five jumps away from the mouse. The cat jumps once while the mouse takes three steps and one cat-jump is 10 mouse-steps long. Assume the positions of the hole, mouse and cat form a line with the mouse between the hole and the cat. Will the cat catch the mouse? If not, how close does it come?

Problem Set 15

1. Consider all five digit numbers whose first two digits are the same and last two digits are the same. Find the smallest and largest among them that are divisible by 89.

2. There are two cubical dice, but one has a blank face instead of a 1 and the other has a blank face instead of a 4. What is the probability of getting a sum of 7 when the pair is rolled?

3. Kathy plans to travel from Andover to Boston and arrive at noon. If her speed for the first half of the distance was 21/23 of her planned speed, at what fraction of her planned speed must she travel for the remainder of the trip to arrive at noon?

4. Sam wants to distribute 174 identical items into a number of boxes. Given that each box can have 12 to 22 items placed in it and that each box will have a different number of items, in how many ways can Sam do this?

5. A positive three-digit integer is selected. The digits are rearranged to form another three-digit integer with no digit in its original position. The positive difference of these two integers is a perfect cube which is less than 100. List the pairs of integers for which this is possible.

6. A six digit number is divisible by 11. All of the digits are different and none are zero. Using only the six digits of this number, how many different numbers divisible by 11 can be formed?

7. In a rectangle ABCD with AB=4 and BC=2, points X, Y, and Z lie on segment CD so that CX=XY=YZ=ZD=1. Segment BZ intersects diagonal AC at P. Find the length of PX.

8. Three overlapping circles in plane each have radius 1 meter. The center of each circle lies on one of the intersection points of the other two. What area is enclosed by the three circles?

9. Point E lies in square ABCD with AE = 4 and BE = 3. What is the length of CE if the sides of the square are 5?

10. Equilateral triangles ABC and ADE are positioned on a coordinate plane so that the coordinates of A, B and D are (0,0), (6,0) and (0,6) respectively. If points C and E have positive coordinates what is the area of the region common to the two given triangles?

11. There are 12 checkers arranged around a circle. Three are red and nine are black. If no two red checkers are adjacent, how many ways can the checkers be arranged around the circle?

12. The sum of three numbers is zero and the sum of their cubes is 12. Find the product of the three numbers.

13. What is the largest two-digit number with exactly 12 positive divisors (including itself and 1)?

14. In triangle ABC side AB = 25 while altitude BE = 24 and altitude AD = 20. What is the perimeter of triangle ABC?

15. There are two cubical dice, but one has a blank face instead of a 2 and the other has a blank face instead of a 5. What is the probability of getting a sum greater than 7 when throwing this pair of dice?

16. What is the probability that a pair of fair dice will have sums of 7, 11 and 2, in some order, on three consecutive rolls of the dice?

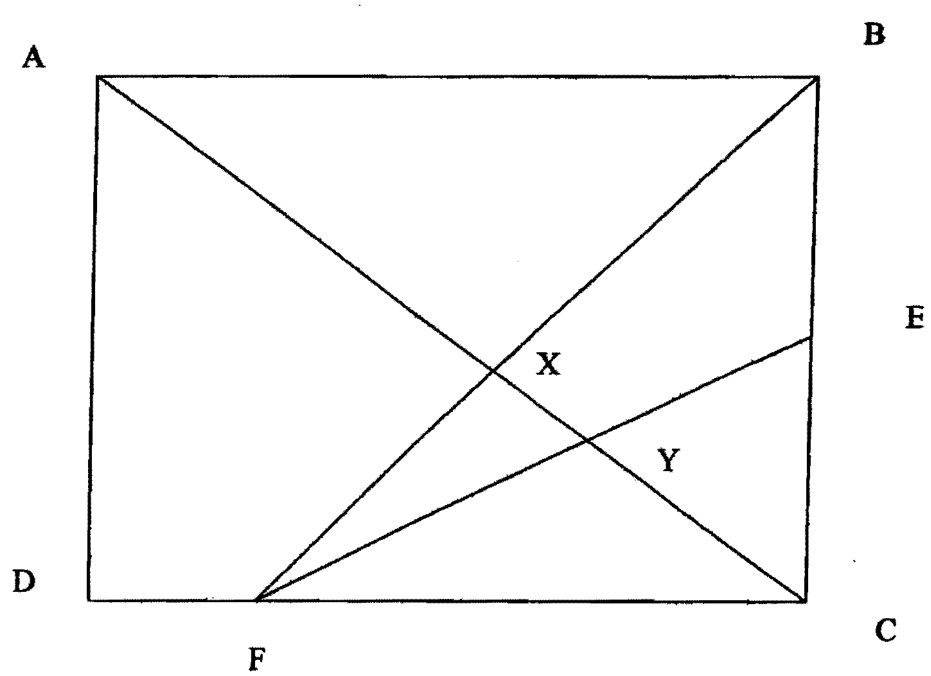
17. What is the unit's digit of $3^{1001}7^{1002}13^{1003}$?

18. A train starts a trip at a uniform rate but after 1 hour must reduce its rate to 2/3 of the original rate. It arrives at its destination 37 minutes late. If it could have traveled an addition 10 miles before reducing its rate it would have arrived only 32 minutes late. What was the original rate of the train?

19. There are four integers, (1,A,B,C), so that $A + B + C = 2001$ and $1 < A < B < C$. From these four it is possible to select six combinations of two numbers each. The sums of these six pairs form an arithmetic progression. What are the values of A, B and C?

20. The ten integers 1, 2, 3, ..., 10 are arranged along the circumference of a circle so that the sum of any two adjacent integers is not a multiple of 3, 5, or 7. List an acceptable order for the integers.

21. In rectangle ABCD, point E is the midpoint of BC and point F lies on CD so that DF:FC = 1:2. Diagonal AC intersect BF at X and EF at Y. What is the ratio AX:XY:YC, expressed in lowest terms?



Problem Set 16

1. Let A be a sequence of integers a_1, a_2, a_3, a_4 and a_5. Let F be a sequence f_1, f_2, f_3, f_4 where $f_n = a_{n+1} - a_n$. Let S be a sequence s_1, s_2, s_3 where $s_n = f_{n+1} - f_n$. If $a_3 = 20$ and $s_1 = s_2 = s_3 = 3$, what is the value of $a_1 + a_5$?

2. When divided by 1990 and 1991, the natural number N gives the same remainder of 77. What is the remainder when N is divided by 22?

3. John and Mary live in the same skyscraper. There are ten apartments on each floor. The apartments are numbered consecutively on the first floor from 1 through 10; on the second floor from 11 through 20; and so on. It is known that the number of John's floor is equal to the number of Mary's apartment. Moreover, the sum of the numbers of their apartments is equal to 239. What is the number of John's apartment.

4. There is a subset of 17 numbers drawn from the set of integers 1 to 100 so that the sum of any pair of them is a multiple of 6. What is the smallest element of the subset?

5. Rectangle ABCD has an area of 12 sq units. Points X and Y lie on CD so that CX = XY = YD and BY and AX intersect at P. What is the area of triangle AYP?

6. How many four-digit positive numbers can be written using exactly 3 different digits in each number? (Example: 7879 uses 3 different digits.)

7. Sally ran 5 miles in one hour. For the first 20 minutes she averaged 4.5 mph. What was her average speed for the remaining 40 minutes?

8. A bag contains 5 red marbles, 5 blue marbles, 5 green marbles and 5 yellow marbles. If 4 are drawn without replacement, what is the probability that at least two are the same color?

9. What is the sum of all the positive 3-digit numbers whose digits are all odd?

10. In triangle ABC points D, E, and F lie on AB, BC, and AC respectively so that AD:DB = 1:2, BE:EC = 1:3 and CF:FA = 1:4. What fraction of the area of triangle ABC lies in triangle DEF?

11. How many numbers of the form $100a + 10b + c$ can be written if $a > b > c$?

12. What is the 100^{th} digit to the right of the decimal place in the decimal expression of 1/74?

13. In right triangle ABC with right angle at C, CA = 6, CB = 8. Lines are drawn parallel to CB and intersect CA at 1 unit intervals and these unit wide strips are alternately shaded blue and red. If the longest strip is blue, what is the ratio of the blue shaded area to the red shaded area?

14. There are two jobs, job A and job B. If the jobs are done individually, Mr. Cheung can finish A in 10 days and B in 15 days., while Mr. Lee can finish A in 8 days and B in 20 days. What is the minimum time to finish both jobs if both men work the same number of days.

15. How many different 5-digit numbers can be written if the product of the digits is 4?

16. A bicycle went up a hill at 10 mph and came down at 20 mph. What was the average speed?

17. A sequence of numbers is generated by letting $a_1 = 3$ and $a_{n+1} = (2a_n - 1)/(a_n + 1)$. What is the 100^{th} term in this sequence?

18. A two-digit number N is divisible by 5. When the digits are reversed and this new number is subtracted from the original the absolute value of the result is a square. How many values are possible for N?

19. Five boys divide a certain number of marbles as follows: Abe takes 1 marble and a fifth of the remaining marbles, the Bill takes 1 marble and a fifth of the remaining marbles. Charlie, Ed and Frank follow the same pattern. What is the minimum number of marbles that must be available at the beginning to make this process possible?

20. Find the remainder when $x^{100} - 2x^{55} + 1$ is divided by $x^2 - 1$.

21. Let y = (1)(2)(3)(4).........(20). What is the sum of the last five digits of y?

22. Arrange the digits 1 through 9 in a row, without repetition, so that:
All the digits between 1 and 2 add up to 6
All the digits between 2 and 3 add up to 14
All the digits between 3 and 4 add up to 38
All the digits between 4 an 5 add up to 9
Find the smallest value for this 9-digit number

23. Consider the number pattern shown.

1
3 5
7 9 11
13 15 17 19
* * * * *

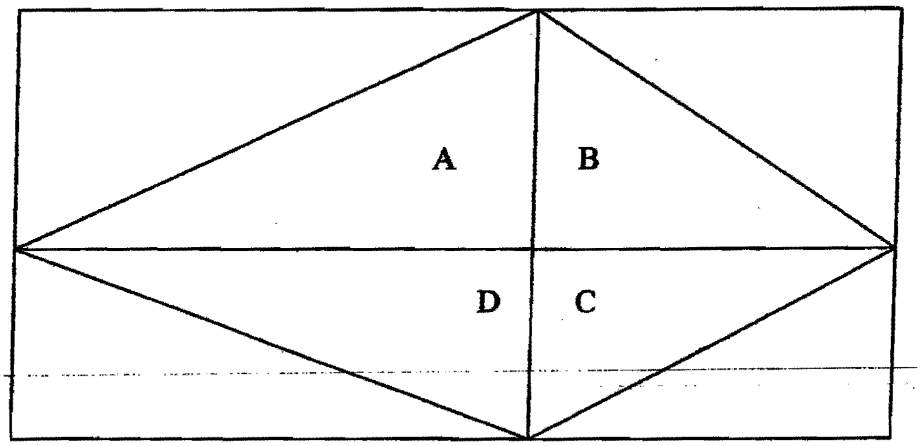
Let A and B be consecutive entries in the i^{th} row and C be an entry in the $(i + 1)^{th}$ row just below A and B. Find the value of C if the sum A + B + C = 2045.

24. Find the total of $\frac{1}{2} + \left(\frac{1}{3} + \frac{2}{3}\right) + \left(\frac{1}{4} + \frac{2}{4} + \frac{3}{4}\right) + \dots\dots\dots + \left(\frac{1}{100} + \frac{2}{100} + \dots + \frac{99}{100}\right)$.

Problem Set 17

1. Two bulbs flash at regular intervals or 30 and 36 seconds. Both bulbs flash together for the first time at 10:45 a.m.. At what time do they flash together for the 15^{th} time?

2. In the figure the areas of the triangles marked as A, B and C are 35, 14 and 11 respectively. What is the area of the triangle marked as D?



3. When X is decreased by 3 or increased by 69 both results are perfect squares. List all the possible integer values of X.

4. Two trains start at the same time, one from A and the other from B, and travel toward each other. If they arrive at B and A 4 hours and 9 hours after they meet, what is the ratio of the speed of the faster train to the speed of the slower train?

5. When 57, 96 and 161 are divided by a positive integer K the remainder is the same for all three divisions. What is the greatest possible value of K?

6. A fruit company orders 4800 kg of oranges at $1.80 per kg. The shipping cost is $3000. Suppose 10% of the oranges spoil during shipping and the remaining oranges are sold. What should the selling price per kg be if the company wants to make an 8% profit?

7. In a party of N friends everyone shakes hands with everyone else once. What is the value of N if there are a total of 496 handshakes?

8. A bus starts from town A toward B and at the same time a bus starts from town B toward A. Both travel at constant rates to their destination and back home without stopping. The first time they meet they are 90 km from town A and the second time they meet they are 50 km from town B. What is the distance between town A and town B?

9. A contractor hires two men to build a brick wall. One man can build it alone in 9 hours and the other can build it alone in 10 hours. However, when the two men work together, there is a shortfall of 10 bricks per hour, and it takes them exactly 5 hours to complete the wall. What is the total number of bricks in the wall?

10. Find the last two digits of 6^{2008}.

11. Two sides of a triangle are 2008 and 8002 units. If the third side is an integer measure in the same units, how many different triangles are possible?

12. A combination lock consists of four reels that display the digits from 1 to 9. How many different combinations are possible if the first digit is a multiple of 3, the second digit is a prime, the third digit is even and the four-digit number formed is divisible by 4?

13. On a true/false test of 100 items, every item number that is a multiple of 5 is true, all the others are false. A student marks all the multiples of 4 as false and all the rest as true. How many of the items did the student answer correctly?

14. When the five-digit number 9abcd is divided by 9 the result is dcba9. What is dcba?

15. More than 50 children are seated in a circle and they count around the circle clockwise beginning at 1. The same student counts both 13 and 2019. What is the minimum number of students seated in the circle?

16. Given the set {1,4,7,10,13}, select three of them as values for x,y and z. How many different values are possible for $x + y - z$?

17. Numbers such as 543 and 652 have their digits in strictly decreasing order because each digit is less than the digit to its left. How many such numbers are there between 100 and 999?

18. Find the fraction with the smallest denominator which falls between 97/36 and 96/35.

19. Billy has $1, $5, and $10 bills in his wallet. He has at least one bill of each denomination and a total of 13 bills with a value of $ 77. How many $5 bills does he have?

20. Peter begins counting up form a 100 by 6's (100, 106, ..) and Mary begins counting down from 1000 by 9's (1000, 991, ...) at the same time. If they count at the same rate which number will both say at the same time?

Problem Set 18

1. What is the largest positive integer less than 100 which has exactly 12 positive divisors?

2. A pair of positive integers has a greatest common factor of 12 and a least common multiple of 3000. What is the minimum possible sum of this pair of numbers?

3. Find the 2008th decimal digit when 1/13 is expressed in decimal form.

4. A five-digit perfect square in the form of 4abc9 has $a > b > c$. What is the value of $a + b + c$?

5. Four different digits are used to form four-digit numbers. The sum of the largest and smallest of these is 11359. What is the smallest of these numbers?

6. A train travels from A to B. If it increases its normal speed by 15 km/h it arrives 48 minutes ahead of schedule. If it decreases its normal speed by 10 km/h it arrives 48 minutes late. What is the distance between A and B?

7. Five boxes have different weights, each less than 100 kilograms. The boxes are weighed in pairs for all possible combinations. The weights are 112, 119, 116, 117, 118, 116, 114, 121, 120 and 115 kilograms. What is the weight of the heaviest box?

8. When a certain two-digit number, AB, is added to another two-digit number, BA, the sum is a perfect square. Find the sum of all such possible two-digit numbers.

9. Let a, b, and c be two-digit numbers. The units digit of a is 4, the units digit of b is 8 and the tens digit of c is 3. If $ab + c = 2008$, find the value of $a + b + c$.

10. A class of students purchased some notebooks. If they were all given to the girls in the class each would receive 28. If they were all given to the boys in the class they would each receive 21. If they are distributed equally among all the class members, how many will each student receive?

11. The edge of a cube is 8 cm. All the faces are painted orange. It is then cut into small cubes of edge 1 cm. How many of the small cubes have exactly two faces painted?

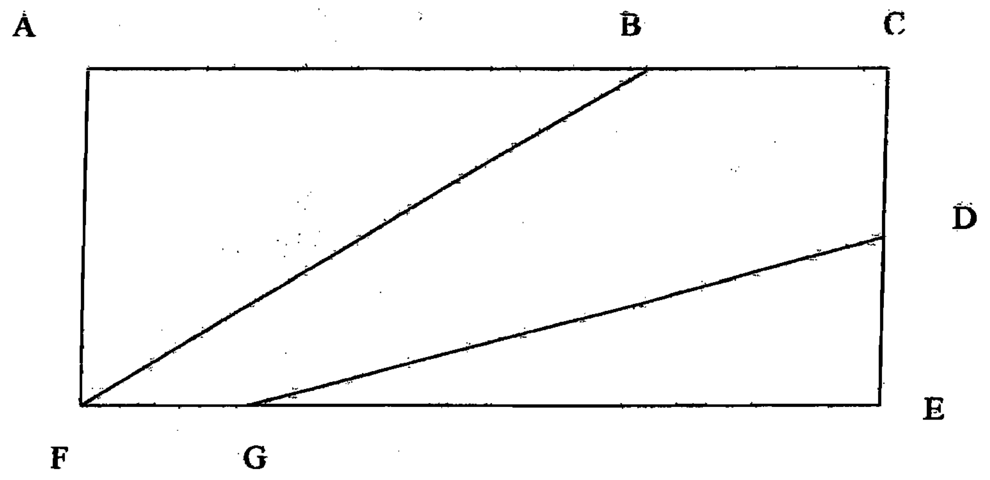
12. By using the integers 1, 2, 3, 4, 5, 6 and 7 without repetition, how many even numbers less than 6000 can be formed?

13. An integer P yields a remainder of 3 when divided by 5, a remainder of 5 when divided by 9 and a reminder of 11 when divided by 13. If P is less than 2000, what is the maximum value of P?

14. How many different isosceles triangles can be formed with a perimeter of 37 where each side is an integer number of units?

15. Lilly plans to spend all of her \$31 to buy types of pens that cost \$2, \$3 and \$4 respectively. If she buys at least one pen of each type, what is the maximum number of pens she can buy?

16. In the rectangle D is the midpoint of $\overline{CE}$, AC = 3$\overline{BC}$ and $\overline{GE}$ = 4$\overline{FG}$. The area of the rectangle is 255 square units. What is the area of the polygon BCDGF?



17. If 2004 is divided by a two-digit number, the remainder is 9. How many such two-digit numbers are there?

18. A positive number when divided by 6, 7 or 8 leaves a remainder of 1. It is divisible by 5. What is the smallest possible value for this number?

19. How many 4-digit positive numbers are there such that the product of their digits is a prime number?

20. All the angles of a hexagon are 120^0 and the lengths of four consecutive sides are, in order, 5, 6, 4 and 9 units. What is the perimeter of the hexagon?