1. 45

If two-thirds of the shed takes 90 minutes to paint, then the remaining one-third will take half as long or 45 minutes to paint.

2. 12000

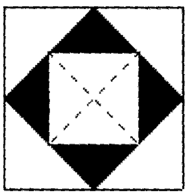
Three hours and 20 minutes is equal to $3(60)+20=200$ minutes which is equal to $200(60)=12000$ seconds

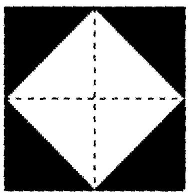
3. 62

If the average height of the 6 Brady Bunch kids is 66 inches, then the combined height of the 6 kids is $6(66)=396$. If the average height of the 8 Eight is Enough kids is 59 inches, then the combined height of the 8 kids is $8(59)=472$. The average height of the 14 kids is their total height divided by 14 which would be $396+472=868$ divided by 14 which is 62 inches.

4. 68

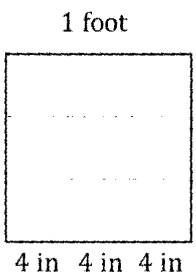
The diagram below on the left shows that the middle sized square is twice the size of the smallest square since it consists of the 4 shaded triangles which are the same size as the four smallest, unshaded triangles. The diagram on the right shows that the largest square is twice the size of the middle square for similar reasons. So the largest square's area is 4 times the area of the smallest which is $4(17)=68$ m^2.





5. 4536

If the dimensions of the deck are 18 ft by 28 ft, then the area of the deck is 504 ft^2. If each tile is 4 in on a side though, it would take 9 tiles to fill one square foot (as shown below). So John will need $9(504)=4536$ tiles.



6. $\underline{\quad 311 \quad}$

$110101_2 = 1(2)^5 + 1(2)^4 + 0(2)^3 + 1(2)^2 + 0(2)^1 + 1(2)^0$

Since we want to change this to base 4 and since 4 is a power of 2, we can change bases without changing to base 10 first which is how we'd do it if we were switching to most bases. The powers of 4 are all the same as the even powers of 2, so let's rewrite that sum of powers like this :

$2(2)^4 + 1(2)^4 + 1(2)^2 + 1(2)^0 = 2(4)^2 + 1(4)^2 + 1(4)^1 + 1(4)^0 = 3(4)^2 + 1(4)^1 + 1(4)^0$

Which is 311 in base 4.

7. $\underline{\quad 6 \quad}$

Since the largest total you can get from rolling three dice is 18, getting a total of 16 would mean the dice were all pretty high numbers (on a scale of 1 to 6).

Getting a total of 16 would mean you were only 2 off of the highest and those 2 could have come from any one of the three dice (3 ways).

Or 2 of the 3 dice could be one less than a 6 with just one of the dice being a 6 (3 ways).

So there must be 6 ways to get a total of 16. If you are unconvinced you could always just list them : (4, 6, 6) ; (6, 4, 6) ; (6, 6, 4) ; (5, 5, 6) ; (5, 6, 5) ; (6, 5, 5).

8. $\underline{\quad 11 \quad}$

We could find a common denominator for 36, 7 and 11, but if you know the decimal equivalents to some common fractions, using decimals might be faster in this case. The fraction $\frac{3}{7} \approx 0.43$ and $\frac{8}{11} \approx 0.73$, so we need something out of 36 to fall between those two values. The fraction $\frac{16}{36} = \frac{4}{9} \approx 0.44$ so that will be the smallest one that fits. The fraction $\frac{27}{36} = \frac{3}{4} = 0.75$ so that would be a little too big. So the possible values for A would be any integer from 16 to 26 for a total of 11 values of A.

9. $\underline{\quad 7 \quad}$

The powers of 13 will have the following units digits : 3, 9, 7, 1, 3, 9, 7, 1, ...

The powers of 17 will have the following units digits : 7, 9, 3, 1, 7, 9, 3, 1, ...

Since both repeat every 4, every multiple of 4 in the exponent will have the same units digit as the 4^{th} power of the original number. So all we really need to do is find the remainder when the exponent is divided by 4 and raise the base to that power. Since 840 is a multiple of 4, the 13^{843} will have the same units digit as 13^3 which is 7.

Similarly 17^{264} will have the same units digit as 2^4 since 264 is a multiple of 4. That units digit would be 1. Finally the product of those units digits would be $7 \times 1 = 7$.

10. 844

If Sam still has 173 of the original 384 feet left to get to the rock, then he has traveled 384 – 173 = 211 feet so far. If it takes Sam 1 minute to travel 3 inches, then it will take him 4 minutes to travel 12 inches (1 foot). So it must have taken Sam 4(211) = 844 minutes to make it to the log.

11. 396

Any positive integer which is one more than a multiple of 5 would have to have a units digit of 1 or 6. Since 360 is a multiple of 9, we'd need to add another multiple of 9 to 360 and have it end it 1 or 6. So we need the smallest multiple of 9 which has a units digit of 1 or 6. The smallest multiple of 9 which ends in 1 is 81 and the smallest which ends in 6 is 36. So the smallest multiple of 9 which is greater than 360 and is one greater than a multiple of 5 must be 360 + 36 = 396.

12. 15

Each of the 4 switches can either be up or down meaning that together there are $2^4 = 16$ possible ways for the switches to be positioned. Since all the switches being down does not turn a light on, there must only be 16 – 1 = 15 different lights the switches can control.

13. 442

Without any restrictions, Jimmy can choose 3 out of the 15 games by computing the combination "15 choose 3" as follows : ${}_{15}C_3 = \binom{15}{3} = \frac{15!}{3!(15-3)!} = \frac{15\cdot14\cdot13}{3\cdot2\cdot1} = 455$. However he can't bring both of his football games. So let's figure out how many of those 455 sets of 3 included both football games and subtract those. If he did choose both football games he could match them up with any of the remaining 13 games. So there are 13 sets which include both football games and 455 – 13 = 442 sets of 3 games that do not.

14. 49

Since each positive integer less than 50 can be multiplied by a particular "number" to get a product of 24, the answer is 49 since there are 49 positive integers less than 50. For example, 37 (a positive integer) could be multiplied by $\frac{24}{37}$ (a number) to get a product of 24. There are exactly 49 of these values to go with the 49 possible integers, specifically $\frac{24}{n}$ where n is any positive integer from 1 to 49.

15. $66\frac{2}{3}$

If they are supposed to divide the inheritance in the ratio 2 : 3 : 4 then the amounts they each get could be expressed as $2x$, $3x$ and $4x$ meaning that Kari gets $\frac{2x}{9x}$ or $\frac{2}{9}$ of the total. Similarly, Robert gets $\frac{3}{9}$ or $\frac{1}{3}$ of the total. Finally Steve gets $\frac{4}{9}$ of the total. When Steve takes his $\frac{4}{9}$ he leaves behind $\frac{5}{9}$ of the total. So when Robert is given $\frac{1}{3}$ of what's left, he is leaving behind $\frac{2}{3}$ of that $\frac{5}{9}$ or $\frac{2}{3} \times \frac{5}{9} = \frac{10}{27}$ of the total for Kari. But Kari is only supposed to get $\frac{2}{9} = \frac{6}{27}$ of the total. So Kari is getting $\frac{4}{27}$ more than she is supposed to which compared to the $\frac{6}{27}$ she's supposed to get is $\frac{4}{6} = \frac{2}{3} = 66\frac{2}{3}\%$ more than she is supposed to receive.

16. 42

To be a perfect square, a number's prime factorization must have all even exponents. $37800 = 2^3 \cdot 3^3 \cdot 5^2 \cdot 7^1$. Since the exponents of 2, 3 and 7 are all odd, we'd have to multiply 37800 by at least $2 \cdot 3 \cdot 7 = 42$ to make it into a perfect square.

17. 60

Normally we would just say that there are $6! = 720$ ways to put 6 things in order. However putting things in a circle is a little different. Let's say we chose an order of ABCDEF for the 6 charms. Well, a second order of BCDEFA would be the same on a circle since in going around the circle in the first arrangement you would find A after getting to F just as you would in the second arrangement. So we would clearly be over counting if we just said $6! = 720$ arrangements. Since you could start going around the circle from any of the 6 charms, each unique arrangement could actually be written in 6 different ways : ABCDEF, BCDEFA, CDEFAB, DEFABC, EFABCD, FABCDE , so we would need to divide 720 by 6 to get 120. If this were arrangements around a table we would be done, but around a bracelet or key chain we still have work to do. Since you can take a bracelet off, flip it over, and put it back on without it being a different arrangement of the charms, we now need to divide by 2 since any order and its reverse (ABCDEF and FEDCBA) would be the same arrangement on the bracelet. That makes a total of $6! \div 6 \div 2 = 60$ arrangements.

18. $6\frac{1}{4}$

The prices were lowered to 80% of original when they were supposed to be lowered to 75% of original. They need to be lowered 5 of the remaining 80 percent. That's a decrease of $\frac{5}{80}$ which equals $\frac{1}{16} = .0625 = 6.25\% = 6\frac{1}{4}\%$.

19. ___199___

In order for a fraction to have a decimal representation which terminates (ends), the denominator's prime factorization must only be of the form $2^x \cdot 5^y$. The reason this is true is that any decimal which terminates can be written as a fraction with a power of 10 as the denominator. Any fraction with a denominator of the form $2^x \cdot 5^y$ can have its numerator and denominator both multiplied by some other number of the form $2^a \cdot 5^b$ so that the equivalent fraction has a denominator that is a power of 10 and can be written as a terminating decimal.

This fraction's denominator, 12600 has a prime factorization of $2^3 \cdot 3^2 \cdot 5^2 \cdot 7^1$. In order to make the fraction terminate, we need to get rid of the $3^2 \cdot 7^1 = 63$ in the denominator by placing a multiple of 63 in the numerator. Since the value of the fraction must be less than 1 we need the multiple of 63 to be less than 12600 which equals $63 \cdot 200$. So any multiple of 63 from $63 \cdot 1$ to $63 \cdot 199$ would work. That gives us 199 values for x.

20. ___90___

Mike needs 2 out of his 4 shirts, 2 out of his 3 jeans and 1 of his 5 pairs of sneakers. He can make those individual decisions in ${}_4C_2 = \frac{4!}{2!(4-2)!} = 6$, ${}_3C_2 = \frac{3!}{2!(3-2)!} = 3$ and ${}_5C_1 = 5$ ways, respectively. To make all those decisions together he can do it in $6 \cdot 3 \cdot 5 = 90$ ways.

21. ___24___

The more bears there are, the more leaves that will get eaten which makes the number of bears and the number of leaves directly proportional. The more minutes there are, the more leaves that will get eaten which also makes the number of minutes and the number of leaves directly proportional. Since if there are more bears it would takes less minutes to eat a given number of leaves, the number of bears and the number of minutes are inversely proportional.

The quotient of directly proportional things will remain constant and the product of inversely proportional things will remain constant. So the equation below will be true.

$\frac{\text{bears} \times \text{minutes}}{\text{leaves}} =$ a constant or in this specific case, $\frac{12 \times 16}{24} = c = 8$.

Using the second set of information $8 = \frac{18 \times m}{54}$ so $m = 24$ minutes.

22. $\underline{\quad -1 \quad}$

The slope (m) of the line that passes through 2 points (x_1, y_1) and (x_2, y_2) is found by the formula $= \frac{y_2 - y_1}{x_2 - x_1}$. We can fill in what we know, giving us $\frac{2}{3} = \frac{10-4}{8-n}$. Solving that equation gives us :

$$2(8-n) = 3(6)$$
$$16 - 2n = 18$$
$$-2n = 2$$
$$n = -1$$

23. $\underline{\quad 50 \quad}$

The first several terms of this sequence are as follows :

1, 3, 7, 15, 31, 63, 127, 255, 511, ...

Just looking at the sequence, every other term is divisible by 3. While we've looked at enough terms to convince most people that every other number will be divisible by 3, we should probably prove it to ourselves.

Let's look at the remainders when the numbers in the list are divided by 3 :

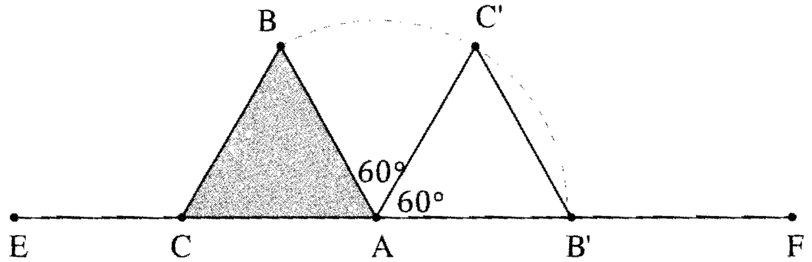
1, 0, 1, 0, 1, 0, 1, 0, 1, 0, ...

That just shows the same thing, that every other number is divisible by 3, but why? When a number 1 greater than a multiple of 3 (like 1) is doubled, you get a number 2 greater than a multiple of 3, and then when you add 1 you get a number 3 more than a multiple of 3 which is itself a multiple of 3. Then, when you double a multiple of 3 and add 1 you get a number one more than a multiple of 3, so this pattern repeats every other term as we thought it would,

Therefore 50 of the 100 terms would be divisible by 3.

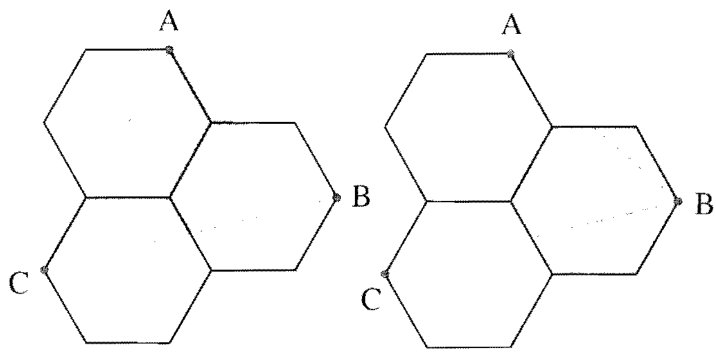
24. $\underline{\quad 13 \quad}$

As the segment AB rotates around the end point A, the point B will travel in a circle with radius equal to the length of the segment. Since triangle ABC is equilateral, angle CAB is 60° as is angle B'AC' meaning that angle BAC' is 60° as well. Point B travels along the circular path of radius 6 cm equivalent to one-third of the circle. (one-third because the measure of angle BAB' is 120° of the 360° in a full circle) The circumference of a circle with radius 6 cm would be 12π cm and one-third of that is $4\pi \approx 4(3.14) = 12.56 \approx 13$.



25. $81\sqrt{3}$

The diagram on the left shows the triangle we want to find the area of shaded in. On the right, two sections of the shaded triangle are unshaded, but equally sized areas outside the triangle are shaded instead. So the area of the shaded region in the two pictures is equal. The area shaded in the right diagram is easier to calculate, though, since it consists of one and a half of the hexagons. The area of a regular hexagon can be found with the formula $A = \frac{3\sqrt{3}s^2}{2} = \frac{3\sqrt{3}\cdot 36}{2} = 54\sqrt{3}$. One and a half that area is $81\sqrt{3}$.



26. 210

If you were to randomly select 4 different digits of the 10, no matter which 4 you selected there would be only one way to put them in order from least to greatest. For example if you chose 3, 7, 0, 5 it would yield the phone number (555) 555-0357. So the number of phone numbers in which each digit was bigger than the previous digit (at least for the last 4 digits) is equal to the number of ways you can chose 4 of the 10 digits. That's just ${}_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210$.

27. 12

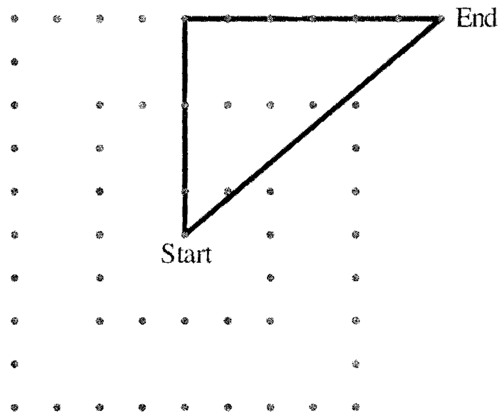
Since Xyla and Yoko want to sit next to each other, let's start by just pretending they are one person and that there are just 3 people that need to be seated in 3 seats. Well that is pretty easy to do, there are just $3! = 6$ ways to seat them. Now that they are seated we need to account for the fact that Xyla and Yoko aren't actually just one person! Since those two could be seated either Xyla/Yoko or Yoko/Xyla, there are two ways to seat them for each of the 6 ways we had so far, so there are $6 \cdot 2 = 12$ ways to seat the 4 friends.

28. $\frac{5}{32}$

There are 2 possibilities when you flip a coin, so there are $2^5 = 32$ ways to flip a coin 5 times in a row. If he only guesses wrong once, we just need to see how many ways we could choose the coin he would guess wrong on. Since there are 5 coins there are only 5 choices. So the probability that he guesses 4 out of 5 right (exactly 1 wrong) is 5 out of 32 or $\frac{5}{32}$.

29. $\sqrt{61}$

The diagram below shows Hao's path along the dotted lines with each point one foot along the path. The darker lines can be used to calculate the distance from Hao's starting and ending points by using the Pythagorean Theorem. The two legs of the triangle are 5 ft and 6ft. Using the Pythagorean Theorem we get :

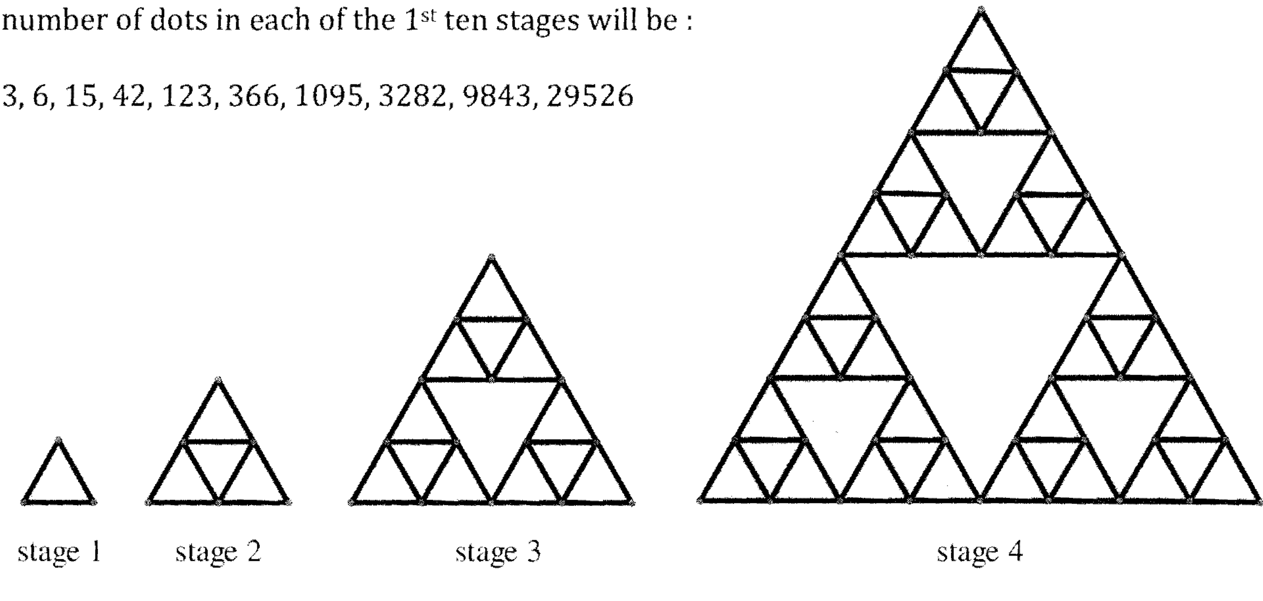


$$5^2 + 6^2 = d^2$$
$$25 + 36 = d^2$$
$$61 = d^2$$
$$\sqrt{61} = d$$

30. 29526

In the 1st stage there are 3 points. In the 2nd stage the diagram contains 3 of the triangles from the 1st stage (shaded), so there are 3 times as many points, but 3 of the points are shared by 2 of the triangles, so the number of points is 3 times as many as the previous stage minus the 3 that count twice. In the 3rd stage, the same thing happens. There are 3 of the 2nd stage formations, but 3 of the points are double-counted, so again there are 3 less than 3 times as many points as in the previous stage. That will continue, so the number of dots in each of the 1st ten stages will be :

3, 6, 15, 42, 123, 366, 1095, 3282, 9843, 29526



1. 25

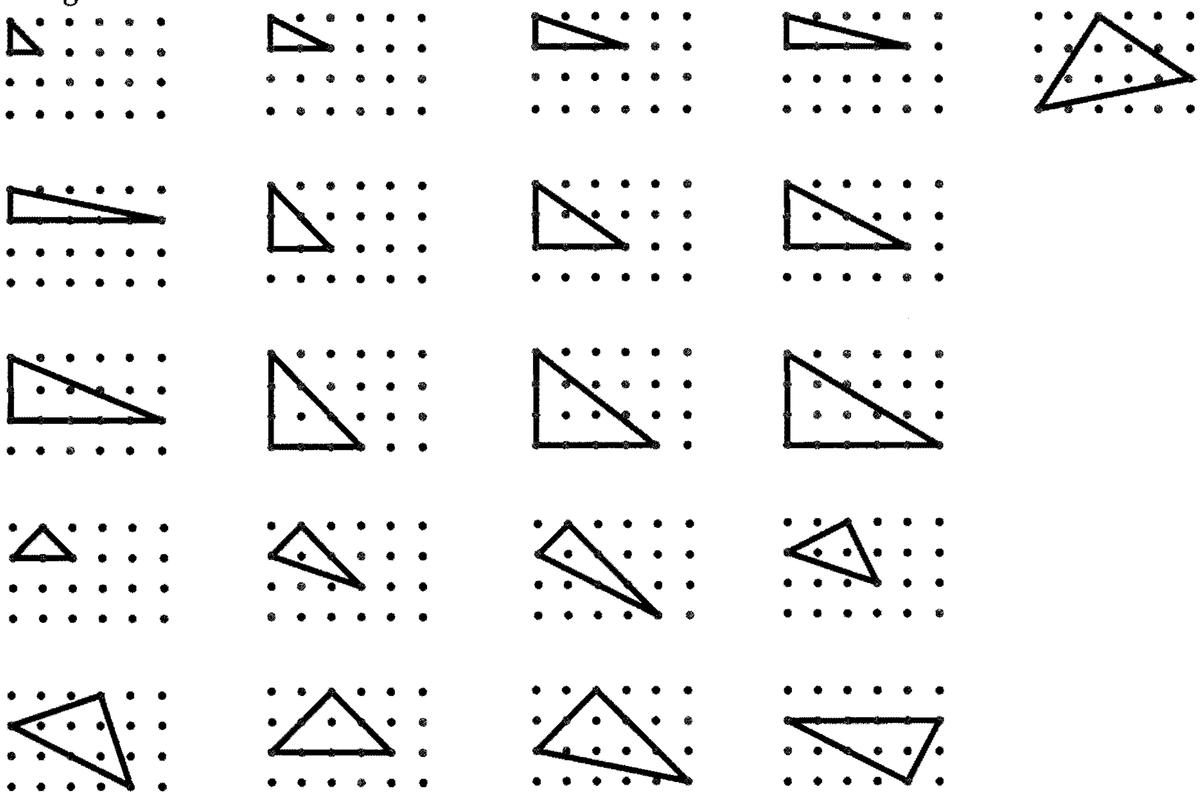
The only way to know how many prime numbers are in any given range is to actually list and count them, so here's the list of prime numbers from the first 100 natural numbers :

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

That's 25 of the first 100 numbers, so 25%.

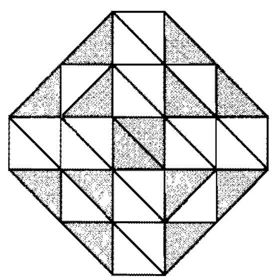
2. 21

The diagram below shows the 21 different triangles you can make using the dots in the grid as vertices. Since you wouldn't have dot paper in a competition you'll want to list the lengths of the legs of the triangles and count the different possible pairs of leg lengths.



3. 41.2

By drawing segments to divide each whole square in half, we can divide the entire picture into several congruent isosceles right triangles. There are 34 triangles and 14 of them are shaded. That's $\frac{14}{34} = \frac{7}{17} \approx 41.2\%$.



4. 240

First let's count the number of ways we can choose the two people who tie. There are ${}_5C_2 = \frac{5!}{2!(5-2)!} = \frac{5 \cdot 4}{2 \cdot 1} = 10$ pairs of people that could tie. Since everyone else finishes in a different time there are 4 different finishing times. The 4 different finishing times can be assigned in $4! = 24$ ways. That means there are a total of $10 \cdot 24 = 240$ different orders they can finish with 2 of the friends tying.

5. 1296

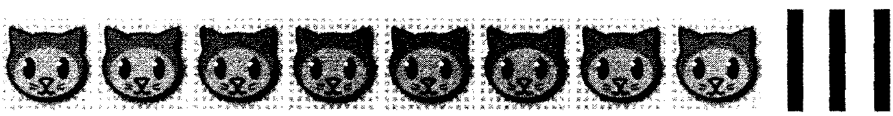
If the side lengths of a rectangular prism were labeled $l, w,$ and h, then the areas of the faces would be lw, lh and wh. Multiplying those areas together would be $l^2w^2h^2 = (lwh)^2$ which is the square of the volume since volume of a rectangular prism is lwh. Since we know the areas of the faces are 48, 54 and 72, then the product of those is $48 \cdot 54 \cdot 72$ and that is the square of the volume. So the volume of the prism would be the square root of that product which equals :

$$\sqrt{48 \cdot 54 \cdot 72} = \sqrt{(16 \cdot 3)(3 \cdot 18)(18 \cdot 4)} = \sqrt{16(3 \cdot 3)(18 \cdot 18)(4)} = 4 \cdot 3 \cdot 18 \cdot 2 = 432$$

If one of the edges was doubled, then the volume would be doubled. If one edge was tripled, the volume would be further tripled. If one edge was cut in half, the volume would be cut in half. It would not matter which edge was altered in which way. The new volume would then be : $432 \cdot 2 \cdot 3 \cdot \frac{1}{2} = 1296$.

6. 165

To divide the 8 kittens into 4 groups we could draw out the 8 kittens and then place 3 dividers among the kittens. Counting the number of ways to place the dividers among the kittens would be one way to count how the kittens were divided. If we line up the kittens and the dividers as shown below :



These 11 things could be arranged in $11! = 39916800$ ways, but we aren't interested in which cats went to which family, and the three dividers are indistinguishable, so we are over counting a lot! Since there are 8 identical cats we need to divide by the $8! = 40320$ ways they could be arranged. We also need to divide by the number of ways the dividers could be interchanged which is $3! = 6$. So the actual number of distinguishable ways the cats could have been adopted is $\frac{11!}{8! \cdot 3!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$.

You could also think of this as there being 11 spots in which you could either place a kitten or a divider. There are ${}_{11}C_3 = \frac{11!}{8! \cdot 3!} = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = 165$ ways in which to choose 3 of the 11 spots to place the dividers and place the kittens in the remaining spots.

7. $\frac{7}{8}$

When the large cube is divided up and painted, it is equally likely that any side of any small cube would end up being the one that was on top of the small cube that is rolled. If we find out how many of the small faces are painted orange and how many small faces there are in total we get the probability that the top face of the rolled cube is orange and then subtract that from 1 to get the probability that the top of the rolled cube is unpainted (not orange). The number of small faces in total is $6 \cdot 8 \cdot 8 \cdot 8$. The number of small faces that are painted orange is $6 \cdot 8 \cdot 8$. The probability that the top face comes up orange then is $\frac{6 \cdot 8 \cdot 8}{6 \cdot 8 \cdot 8 \cdot 8} = \frac{1}{8}$. So the probability the top face is not orange is $\frac{7}{8}$.

8. $\frac{12}{35}$

There are 4 different ways the choices could turn out in terms of even (E) and odd (O) : EEE, EEO, EOO, OOO if we aren't considering the order of the picks. In the first case, EEE, both the sum and product would be even. In the third case, EOO, both the product and the sum would be even. In the fourth case, both the sum and product would be odd. So the only case in which the sum and product don't end up the same in terms of odd and even (parity) is the third case, EEO. Since we are only looking for the difference in the probabilities and this case is the only difference, we only actually need to calculate this case's probability. The probability that 2 evens and 1 odd number are chosen can be found by finding the number of ways 2 of the 3 even numbers can be chosen times the number of ways 1 of the 4 odd numbers can be chosen all over the number of ways 4 of the numbers can be chosen.

"2 out of the 3 even numbers are chosen" can happen in : ${}_3C_2 = \frac{3!}{2!(3-2)!} = 3$ ways.

"1 out of the 4 odd numbers are chosen" can happen in : ${}_4C_1 = \frac{4!}{1!(4-1)!} = 4$ ways.

"3 out of the 7 numbers are chosen" can happen in : ${}_7C_3 = \frac{7!}{3!(7-3)!} = 35$ ways.

So the probability that 2 even numbers and 1 odd number are chosen when 3 of these 7 numbers are chosen would be $\frac{3 \cdot 4}{35} = \frac{12}{35}$.

1. 13996

The given formula basically says that each term is the sum of the terms on either side of it divided by 3. We can rewrite the formula like this : $t_{n+1} = 3t_n - t_{n-1}$ so that we can directly find the next term from the previous two. Since we know the 1st and 2nd terms we can use the formula to find the 3rd term as follows and each later term as follows :

$t_3 = 3t_2 - t_1 = 3(10) - 12 = 18$

$t_4 = 3t_3 - t_2 = 3(18) - 10 = 44$

$t_5 = 3t_4 - t_3 = 3(44) - 18 = 114$

$t_6 = 3t_5 - t_4 = 3(114) - 44 = 298$

$t_7 = 3t_6 - t_5 = 3(298) - 114 = 780$

$t_8 = 3t_7 - t_6 = 3(780) - 298 = 2042$

$t_9 = 3t_8 - t_7 = 3(2042) - 780 = 5346$

$t_{10} = 3t_9 - t_8 = 3(5346) - 2042 = 13996$

2. $\frac{1}{16}$

In order for his pencil to end up back on point A, one of the following sequences of flips would have to occur : HHTT, HTHT, HTTH, THHT, THTH, TTHH

In order for his pencil to end up on point C, one of the following sequences of flips would have to occur : HHHT, HHTH, HTHH, THHH, TTTT

So there are 6 ways to end up on A and 5 ways to end up on C. Since there are 2 ways to flip a coin and the coin is flipped 4 times, there are $2^4 = 16$ ways to flip the coin 4 times. So the probability that he'll end up back on A is $\frac{1}{16}$ greater than him ending up on C.

3. $\frac{263}{325}$

Finding the probability that the two colors are different can be done by finding the probability that the colors would be the same and then subtracting that from 1. To find the probability that the two candies are the same color we can separately find the probability that the two candies are a particular color and add those together.

P(both red) $= \frac{8}{26} \cdot \frac{7}{25} = \frac{56}{650}$

P(both orange) $= \frac{6}{26} \cdot \frac{5}{25} = \frac{30}{650}$

P(both green) $= \frac{4}{26} \cdot \frac{3}{25} = \frac{12}{650}$

P(both yellow) $= \frac{5}{26} \cdot \frac{4}{25} = \frac{20}{650}$

P(both purple) $= \frac{3}{26} \cdot \frac{2}{25} = \frac{6}{650}$

P(both the same color) $= \frac{56}{650} + \frac{30}{650} + \frac{12}{650} + \frac{20}{650} + \frac{6}{650} = \frac{124}{650} = \frac{62}{325}$

P(two different colors) $= 1 - \frac{62}{325} = \frac{263}{325}$

4. $\frac{4}{3}$

In hexagon ABCDEF with side length s, we can connect vertices which have one vertex between them such as segment BF. Using the fact that a regular hexagon can be divided into 6 equilateral triangles as in figure 2, we can see that segment BF would divide two of those triangles in half, making segment AG in figure 1 have a length of $\frac{s}{2}$. Using the Pythagorean Theorem, or knowledge of 30-60-90 triangles, we can find that segment BG has a length of $\frac{\sqrt{3}s}{2}$. That means that segment BF would have a length twice that which is $\sqrt{3}s$.

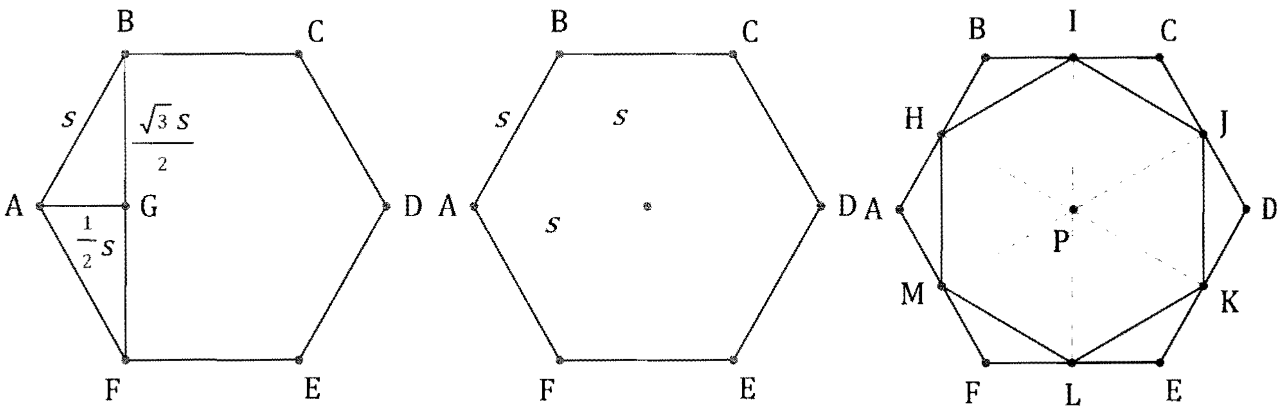


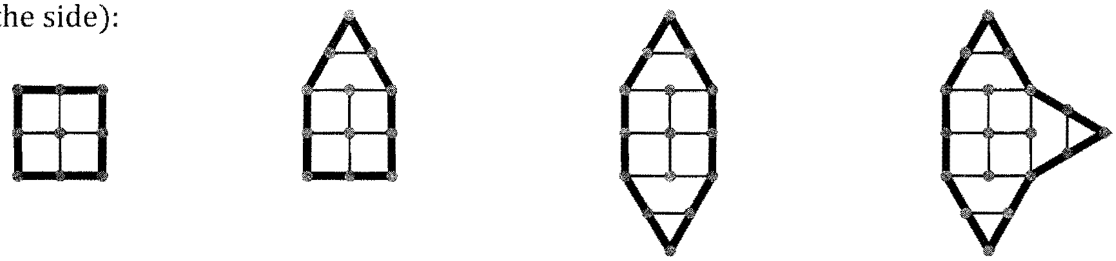
Figure 1 Figure 2 Figure 3

In Figure 3, the hexagon which uses the midpoints of the sides of hexagon ABCDEF is shown, as well as how this new hexagon, HIJKLM, can also be broken up into 6 equilateral triangles. Note that segment IL in Figure 3 is the same length as segment BF $(\sqrt{3}s)$ in figure 1 and that segment IP is the same length then as segment BC $\left(\frac{\sqrt{3}s}{2}\right)$. Since segment IP is a side of one of the equilateral triangles in figure 3, it is the same length as segment HI, also a side of an equilateral triangle as well as being a side of the smaller hexagon. So now we have the ratio between the side lengths of the two hexagons. The ratio of AB to HI is s to $\frac{\sqrt{3}s}{2}$ which can be simplified as follows: $\frac{s}{\frac{\sqrt{3}s}{2}} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$.

The ratio of the areas of two similar figure is always the square of the ratio of the side lengths of those figures. This means the ratio of the areas is $\left(\frac{2}{\sqrt{3}}\right)^2 = \frac{4}{3}$.

5. 411

The triangular numbers are the sum of the consecutive natural numbers starting at 1. All the other figurate, or polygonal, numbers can be found based off the triangular numbers. In the chart below, each number is found by adding the number directly above it to the triangular number before it. For example, the 6th hexagonal number (66) can be found by adding the 6th pentagonal number (51) and the 5th triangular number (15). Showing why this is true would take an awful lot of pictures, but start on your own by trying this. Draw a square array of 9 dots. This is considered the 3rd square number. By placing a triangular array of 3 dots (this is the 2nd triangular number) above the top of the square (3rd square number) so that this array forms a larger equilateral triangle with the top 3 dots from the square, you've built the 3rd pentagonal number. Continuing this as shown below gives the 3rd hexagonal number (by adding that 2nd triangular number on the bottom) and the 3rd heptagonal number (by adding that 2nd triangular number on the side):



9 dots　　12 dots　　15 dots　　18 dots

In this case we keep adding the 2nd triangular number to the 3rd of each successive polygonal number to get the 3rd of the next polygonal number. This can be repeated again and again to represent any polygonal number.

	1st	2nd	3rd	4th	5th	6th	7th	8th
Triangular	1	3	**6**	10	15	21	28	36
Square	1	4	9	**16**	25	36	49	64
Pentagonal	1	5	12	22	**35**	51	70	92
Hexagonal	1	6	15	28	45	**66**	91	120
Heptagonal	1	7	18	34	55	81	**112**	148
Octagonal	1	8	21	40	65	96	133	**176**

So the sum of the 3rd triangular, 4th square, 5th pentagonal, 6th hexagonal, 7th heptagonal number and 8th octagonal number is $6 + 16 + 35 + 66 + 112 + 176 = 411$.

6. 0.154

The probability of spinning a 3 is 20% or 0.2. Therefore the probability of not spinning a 3 would be 80% or 0.8. Since we want 7 kids to spin a 3, and 23 to not spin a 3 the probability of that happening in that order would be $(0.2)^7(0.8)^{23}$. However that would only be the probability that the first 7 kids spun a 3. The number of ways that 7 out of 30 kids could spin a 3 is ${}_{30}C_7 = \frac{30!}{7!(30-7)!} = 2{,}035{,}800$. So the probability that exactly 7 of 30 kids would spin a 3 would be $(0.2)^7(0.8)^{23}(2035800) \approx 0.1538 \approx 0.154$.

7. <u>9116</u>

The formula for finding the sum of an arithmetic series is :

Sum = (1st term + last term) × (number of terms)÷ 2.

If we call the first term y, we can find that the 129th, or last, term is $y + 128(3)$ since the sequence increases by 3 each term for 128 terms after the first. So we can use the formula to find y as follow :

$$26961 = \frac{[y + y + 3(128)] \cdot 129}{2}$$
$$53922 = (2y + 384)129$$
$$418 = 2y + 384$$
$$34 = 2y$$
$$17 = y$$

So the first term is 17 and the sequence continues like this : 17, 20, 23, 26, 29, ...
In the sequence we are looking for the sum of, the 1st term is the 3rd term of the original sequence, so the 1st term is 23. The last term is the same as the last term of the first sequence, so it is $y + 384 = 17 + 384 = 401$. Since there are one-third as many terms in this new sequence, there are 43 terms. So the sum of the terms in the sequence is :
$x = \frac{(23+401)43}{2} = \frac{424 \cdot 43}{2} = 212 \cdot 43 = 9116.$

8. <u>5976</u>

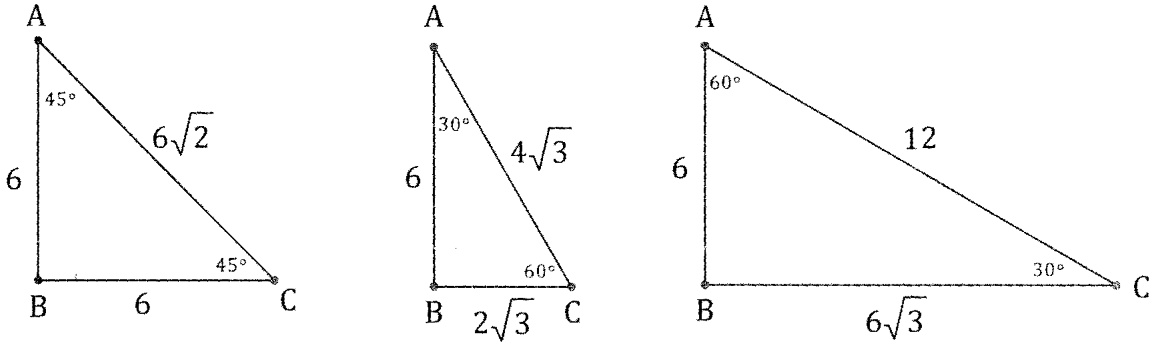
If we are only considering whether a number is divisible by 3 or not, there are only two possibilities for each digit. Either the digit is divisible by 3 (Y) or not divisible by three (N). Since there are 4 digits, there are $2^4 = 16$ cases in terms of which digits are divisible by 3 or not. As it turns out 8 of them have at least 2 consecutive digits which are divisible by 3 and 8 cases in which that does not occur. The 8 cases which do not have at least 2 consecutive digits which are divisible are below along with the number of ways each can occur. Remember that there are 4 digits which are multiples of 3 (0, 3, 6, 9) although 0 cannot be the first digit.

NNNN $= 6 \cdot 6 \cdot 6 \cdot 6 = 1296$
YNNN $= 3 \cdot 6 \cdot 6 \cdot 6 = 648$
NYNN $= 6 \cdot 4 \cdot 6 \cdot 6 = 864$
NNYN $= 6 \cdot 6 \cdot 4 \cdot 6 = 864$
NNNY $= 6 \cdot 6 \cdot 6 \cdot 4 = 864$
NYNY $= 6 \cdot 4 \cdot 6 \cdot 4 = 576$
YNNY $= 3 \cdot 6 \cdot 6 \cdot 4 = 432$
YNYN $= 3 \cdot 6 \cdot 4 \cdot 6 = 432$

The total number of positive 4-digit integers which have no two consecutive digits that are multiples of 3 is $1296 + 648 + 864 + 864 + 864 + 576 + 432 + 432 = 5976$.

9. 10

In the diagram below, the three possible ways which triangle ABC could be drawn are illustrated. The other side lengths of the triangle were determined using the Pythagorean Theorem or general knowledge of 45-45-90 and 30-60-90 triangles, but to explain here would give too much detail.



The result of the three figures is that the hypotenuse of the triangle could end up being either $6\sqrt{2}$, $4\sqrt{3}$ or 12. In other words the hypotenuse could be either $\sqrt{2}$, $\frac{2\sqrt{3}}{3}$ or 2 times as big as the starting leg of the triangle. Since we are going to repeat this process three times there are as many as $3^3 = 27$ possible side lengths for the hypotenuse (AE) of the third triangle(ADE). Some lengths will repeat though so we have to go through and check.

Possible lengths of the first hypotenuse (AC) : $6\sqrt{2}$, $4\sqrt{3}$ and 12

Multiplying each of those by $\sqrt{2}$, $\frac{2\sqrt{3}}{3}$ and 2 gives the possible lengths for the 2nd hypotenuse (AD) as : $12, 4\sqrt{6}, 12\sqrt{2}, 4\sqrt{6}, 8, 8\sqrt{3}, 12\sqrt{2}, 8\sqrt{3}, 24$ although $4\sqrt{6}$, $12\sqrt{2}$ and $8\sqrt{3}$ are listed twice each, so the possible side lengths for segment AD are :

$$12, 4\sqrt{6}, 12\sqrt{2}, 8, 8\sqrt{3} \text{ and } 24$$

To get the possible lengths of the 3rd hypotenuse (AE) we need to multiply each of those 6 possible lengths of the 2nd hypotenuse by $\sqrt{2}$, $\frac{2\sqrt{3}}{3}$ and 2 to get :

$$12\sqrt{2}, 8\sqrt{3}, 24, 8\sqrt{2}, 8\sqrt{6}, 24\sqrt{2}, 8\sqrt{3}, 8\sqrt{2}, 8\sqrt{6}, \frac{16\sqrt{3}}{3}, 16, 16\sqrt{3}, 24, 8\sqrt{6}, 24\sqrt{2}, 16, 16\sqrt{3}, 48$$

Several values in that list appear more than once though. Eliminating the values which repeat leaves us with the following 10 possible lengths of segment AE :

$$12\sqrt{2}, 8\sqrt{3}, 24, 8\sqrt{2}, 8\sqrt{6}, 24\sqrt{2}, \frac{16\sqrt{3}}{3}, 16, 16\sqrt{3}, 48$$

10. 0.95

To find the area of the circle we need to know the radius of the circle. The diameter of the circle would be just as helpful and in this case the diameter of the circle is the distance between the parallel lines. Since parallel lines are always the same distance apart, we just need to find the distance between the parallel lines at any point in the diagram.

Start by drawing a perpendicular line from point E to $\overline{DF}$ at point G. This will be the same length as the diameter of the circle. It also is one side of a right triangle which may be easier to find.

Since the area of the square is 9, the sides of the square must be 3. If $m\angle ABE = 30°$ then $m\angle AEB = 60°$ (since the sum of the angles in a triangle must be 180°) making triangle ABE a 30-60-90 triangle. Since the length of segment AB is 3, the length of segment AE must be $\sqrt{3}$ as this is always true in 30-60-90 triangles. Since the length of segment AE is $\sqrt{3}$, the length of segment ED (the rest of the side of the square) must be $3 - \sqrt{3}$.

Since $\overline{BE} \parallel \overline{DF}$ segment AD is a transversal of those parallel lines and angles AEB and EDF are corresponding angles and therefore congruent. So $m\angle EDG = 60°$ as well and $m\angle DEG = 30°$ making triangle DEG a 30-60-90 triangle as well. Knowing that, the shorter leg of the triangle will be half the hypotenuse, or in this case $\frac{3-\sqrt{3}}{2}$, and the longer leg will be equal to the shorter leg times $\sqrt{3}$, or in this case $\frac{3\sqrt{3}-3}{2}$.

Remember that the length of segment EG is the same length as the diameter of the circle, so we are almost done. The radius of the circle would be half that length, or $\frac{3\sqrt{3}-3}{4}$.

To get the area of the circle we need to calculate πr^2 which in this case is approximately 0.947 or 0.95.