1. 65

If Mayukha wins 7 of the games then she does not win $20 - 7 = 13$ of them. That means she does not win 13 out of 20 which is equivalent to 65 out of 100 or 65%.

2. 60

If 80 degrees is twice another angle, then the other angle is 40 degrees. Since the sum of the three angles must be 180 degrees in any triangle, the third angle must be $180 - (80 + 40) = 180 - 120 = 60$.

3. 6

In order to take the lead, Buffalo needs more points than New England. $45 - 3 = 42$ more points would make it a tie, so they need 43 more points. Even if they scored 8 points every touchdown, 5 touchdowns would only give them 40 points and a 6th touchdown would be needed.

4. $7\frac{3}{8}$

$$10\frac{1}{4} - 2\frac{7}{8} = 10\frac{2}{8} - 2\frac{7}{8} = 9\frac{10}{8} - 2\frac{7}{8} = 7\frac{3}{8}$$

5. 50

Alec does the following even numbered floors : 2, 4, 6, 8... 92. Count those by dividing them all by 2 and you get 1, 2, 3, 4...46. Doing that changes the number of each floor, but not how many floors, and now the numbers clearly indicate Alec is doing 46 floors.

Dan does the following odd numbered floors : 97, 99, 101...191. Count those by adding 1 to each (98, 100, 102...192) then dividing them all by 2 (49, 50, 51...96) then subtracting 48 from each (1, 2, 3...48). That's 48 floors.

Chuck does the following evens : 94, 96, 98...192. Count those by dividing them all by 2 (47, 48, 49...96) then subtracting 46 from all of them (1, 2, 3...50). That's 50 floors.

Justin does the following odds : 1, 3, 5, ...95. Count those by adding one to all of them (2, 4, 6, ...96) and then dividing them all by 2 (1, 2, 3, ...48). That's 48 floors.

So 50 floors is the most anyone was assigned.

6. 30

The ratio of Tom's height to Tom's shadow must be the same as the ratio of Tim's height to Tim's shadow, so if T = Tim's shadow, the following proportion must be true. $\frac{6}{15} = \frac{5}{T}$. Cross multiplying we get 6T = 75 and T = 12.5 feet. That's 2.5 feet shorter than Tom's shadow or 2.5(12) = 30 inches shorter.

7. - 64

$x = -2^5 + (-2)^5 = -32 + (-32) = -64$
$y = -2^6 + (-2)^6 = -64 + 64 = 0$
$x + y = -64 + 0 = -64$

*Remember that $-2^6 = -2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ as the exponent is considered before the negative. However putting the negative in parenthesis with the 2 makes a big difference as $(-2)^6 = (-2)(-2)(-2)(-2)(-2)(-2)$.

8. 49

Distance = Rate × Time

Distance is what we want, the Rate is 70 mph, and the time is 42 minutes or $\frac{7}{10}$ of an hour. So D = $(70)\left(\frac{7}{10}\right) = 49$ miles.

9. 126

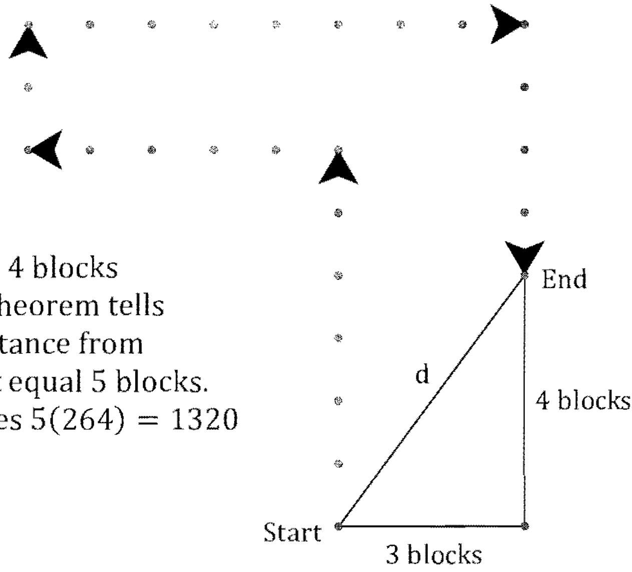
If we call the number of minutes in the movie m, then this equation will give us the value of m.

$$\frac{1}{4}m + \frac{1}{3}m + \left(\frac{1}{4}m - 10\right) + 31 = m$$
$$\frac{5}{6}m + 21 = m$$
$$21 = \frac{1}{6}m$$
$$126 = m$$

10. 96

If 24 is the GCF of 72 and z, then clearly 24 is a factor of z. Therefore, z is a multiple of 24. Likewise, z is a multiple of 32. If we want the smallest value of z, we are looking for the smallest number that is both a multiple of 24 and 32. In other words we want LCM(24, 32) = 96.

11. 1320



The diagram to the right shows the path Adrienne took with each dot being another block and the arrows showing the direction. In the end, Adrienne ends up 3 blocks East and 4 blocks North of where she began. The Pythagorean Theorem tells us that $d^2 = 3^2 + 4^2$, so d (the straight line distance from her apartment to her friend's apartment) must equal 5 blocks. Since each block is 264 feet long, her friend lives $5(264) = 1320$ feet away.

12. 208

The packing box that the gift box was packed in has a volume of $5 \times 8 \times 10 = 400$ in^3. The gift itself is in a box with a volume of $4 \times 6 \times 8 = 192$ in^3. So the remaining volume in the packing box is $400 - 192 = 208$ in^3.

13. 66

Since the triangles must have integral side lengths, the smallest would have side length 1 and a perimeter of $3(1) = 3$. The largest would need to have side lengths less than one-third of 200. The largest such side length would be 66 since $67(3) = 201$ which is too big. So the triangles could have any positive integer side length from 1 to 66, of which there are 66 possibilities.

14. 12

Since the number has to be even, the units digit must be 2, 4 or 6, so there are 3 choices for the units digit. The tens digit can be any of the 4 digits which were not used as the units digit. That means there are a total of $3 \times 4 = 12$ possible even two-digit integers which could be made using 2 different digits from the set.

15. 231

Counting how many ways to choose 20 out of 22 books could seem like a lot of work, but if we are choosing 20 of the books and only leaving behind (NOT choosing) 2 of the books, it is actually easier to think about the problem as having 22 books and choosing 2 of them (to leave on the shelf). The number of ways to choose 2 of 22 things can be calculated as a combination as follows: ${}_{22}C_2 = \binom{22}{2} = \frac{22!}{2!(22-2)!} = \frac{22 \cdot 21}{2 \cdot 1} = 231$.

16. 3

Let's call the 5 children A, B, C, D and E and the order in which they are in line is ABCDE. Consider this order, EABCD. Kids D, C and B all have the correct child in front of him and A has the correct child behind him. Only E does not have the correct child in front of him as there is no child in front of him. Now look at DEABC. Kids A, B and C are all still doing what they should. Student E now has the right child in front of him as well. It is only D who is not doing what he should. The same is true for the order CDEAB. Continuing that pattern gives us BCDEA, but now A doesn't have the right child behind him and B does not have the right child in front of him. So there are only 3 orders in which the kids could stand where 4 of them are doing the right thing : EABCD, DEABC and CDEAB.

17. 361

If we can find the smallest positive integer which is divisible by 6, 8, 9, 12 and 15, then by adding 1 to that integer we would find the smallest integer greater than 1 which was 1 bigger than a multiple of each of the divisors. To find the least common multiple of 6, 8, 9, 12 and 15 we need to look at the prime factorizations of each : $6 = 2 \cdot 3, 8 = 2^3$, $9 = 3^2, 12 = 2^2 \cdot 3$ and $15 = 3 \cdot 5$. By looking at the prime factorizations for each number we can get the least common multiple by taking the highest power of each prime factor that appears in any of the prime factorizations and then taking the product of those powers. In this case the $\text{LCM}(6, 8, 9, 12, 15) = 2^3 \cdot 3^2 \cdot 5 = 360$. So by adding 1 to get 361 we have the smallest integer greater than 1 which would leave a remainder of 1 when divided by any of the given numbers.

18. 15

If a number is divisible by 3 then the sum of the number's digits must be divisible by 3. The sum of the digits in this five-digit number is 2+N+5+7+N = 2N+14. That sum must be a multiple of 3, and since both 2N and 14 are even the sum must also be even. Therefore 2N+14 must equal 18, 24, 30, 36 ....., so 2N must equal 4, 10, 16, 22, ... and N must equal 2, 5, 8, 11.... Since N is a digit, the possible values are 2, 5 and 8 with a sum of 15.

19. 6

If $\frac{1}{3}$ of his hair is removed each appointment, then $\frac{2}{3}$ of his hair would be left after each appointment. After 1 appointment $\frac{2}{3}$ of his hair is left, after 2 appointments $\frac{2}{3} \cdot \frac{2}{3} = \frac{4}{9}$ of his hair would be left and after 3 appointments $\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$ would be left. All we need to do is continue looking at powers of $\frac{2}{3}$ until we find one which has a decimal equivalent less than .1 (or 10%). Looking at the powers of $\frac{2}{3}$ we find, $\frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \frac{16}{81}, \frac{32}{243}, \frac{64}{729}$... it is clear that $\frac{64}{729}$ is less than $\frac{1}{10}$ and less than 10% of his hair would be left. Since $\frac{64}{729} = \left(\frac{2}{3}\right)^6$ then it would take 6 appointments for him to have less than 10% of his hair left.

20. $\frac{1}{70}$

Since the only way to win is to scratch off the 4 correct squares, there is only one way to win. All we need to know is how many ways could Jack choose 4 of the 8 squares to scratch off. To calculate that we look at the combination "8 choose 4" which we find like this : $\binom{8}{4} = \frac{8!}{4!(8-4)!} = \frac{8\cdot7\cdot6\cdot5}{4\cdot3\cdot2\cdot1} = 70$. That means there are 70 ways to choose 4 of the 8 squares and since only 1 of those would result in a match, the probability that Jack wins is $\frac{1}{70}$.

21. 36π

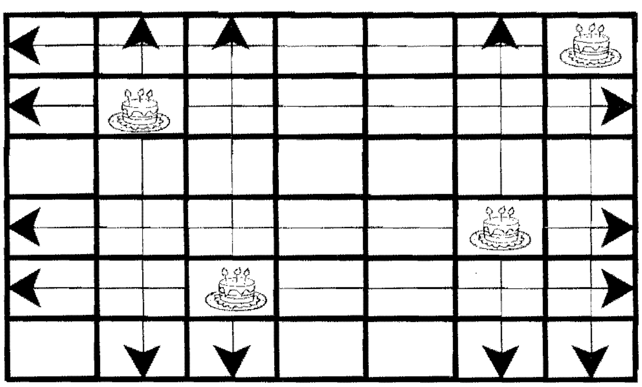
If ten trips around the circle is a distance of 120π, then one trip around the circle is 12π. If the circumference of the circle is 12π, then the diameter of the circle is 12. If the diameter is 12 then the radius is 6 and the area is $\pi \cdot 6^2 = 36\pi$.

22. 2 and 7

If $j(k(b)) = 18$, then we need to find the input for the function j that gives the output of 18. Since $j(x) = x^2 - 7$, we are looking for the value of x such that $18 = x^2 - 7$. Solving that equation we get $x^2 = 25$ and therefore $x = 5$ or -5. These are the values that could be inputted into the j function to give an output of 18, so $k(b) = 5$ or -5. This means that $9 - 2b = 5$ or -5. Solving that equation we get that $-2b = -4$ or -14 and $b = 2$ or 7.

23. 6

If no two stickers are in the same row or column, then the greatest number of rectangles will have at least one sticker in the same row or column as the rectangle in question. Below is one example of how the stickers could be placed leaving only 6 rectangles with no sticker in that rectangle's row or column.



24. 186.40

Of the 18 movies, 8 are matinees and therefore 10 are evening shows. One-third, or 6 of the movies are in 3D, with 5 being evening shows and 1 being a matinee show. That leaves 5 evening shows that aren't in 3D and 7 matinees that aren't in 3D. The two charts below show both the costs and the number of each type of movie seen. The total cost of the movies seen can be found by finding the total from each different priced ticket type and then finding the sum of those totals like this :

Total $= 5(14) + 1(10.40) + 5(10) + 7(8) = 70 + 10.40 + 50 + 56 =$ \$186.40

#	E	M
3D	5	1
reg	5	7

$	E	M
3D	14	10.40
reg	10	8

25. −110

First, we should simplify the expression by noticing that $(11y - 5x)$is the opposite of $(5x - 11y)$, so we could rewrite the expression and then simplify it like this :

$315(5x - 11y) - 30x - 2y + 297(11y - 5x)$

$315(5x - 11y) - 297(5x - 11y) - 30x - 2y$

$(315 - 297)(5x - 11y) - 30x - 2y$

$18(5x - 11y) - 30x - 2y$

$90x - 198y - 30x - 2y$

$60x - 200y$

Now we can substitute in $x = \frac{2}{3}$ and $y = \frac{3}{4}$ to get :

$60\left(\frac{2}{3}\right) - 200\left(\frac{3}{4}\right) = 40 - 150 = -110$

26. $36\pi - 72$

The area of the square is 144. The area of the circle is $\pi \cdot 6^2 = 36\pi$. If 50% of the square is to be painted black, then the area of the black region needs to be 72. Currently the black area is 36π, so the equation $36\pi - x = 72$ would give us the area of the black that needs to be painted white (x). In this case that's $x = 36\pi - 72$.

27. −10

The sum of the solutions to any equation of the form $0 = ax^n + bx^{n-1} + cx^{n-2} + \cdots$ is always $\frac{-b}{a}$. The equation $3x^2 + 30x = 17$ can be rewritten as $0 = 3x^2 + 30x - 17$. The sum of the solutions then would be equal to $\frac{-b}{a} = \frac{-30}{3} = -10$.

28. 24

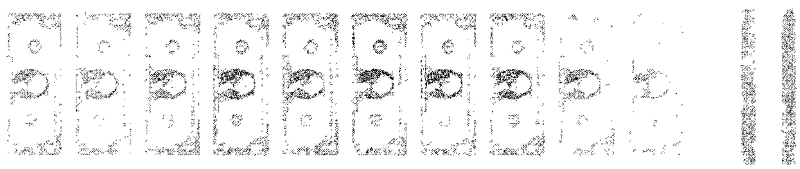
Since we are looking for multiples of 4, this is a good problem to use modular arithmetic on. By looking at the multiples of 3 in modulo 4 as well as looking at the sums in modulo 4, determining which numbers are divisible by 4 is easier (the ones congruent to 0 in modulo 4). Working in modulo 4 basically means we are only looking at the remainder of a number if it was divided by 4. In other words, how much bigger than a multiple of four is the number.

Numbers we are adding : 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, ...
Actual sums : 3, 9, 18, 30, 45, 63, 84, 108, 135, ...
Numbers we are adding written in mod 4 : 3, 2, 1, 0, 3, 2, 1, 0, 3, 2, 1, 0, ...
Sums of the remainders : 3, 5, 6, 6, 9, 11, 12, 12, 15, 17, 18, 18, ...
Sums of the remainders in mod 4 : 3, 1, 2, 2, 1, 3, 0, 0, 3, 1, 2, 2, 1, 3, 0, 0, ...

The sums in mod 4 repeat the pattern 3, 1, 2, 2, 1, 3, 0, 0. This block of 8 would repeat 12 times and each block has two 0s for a total of $12 \times 2 = 24$ numbers divisible by 4. After the 12 blocks the last 4 of the 100 terms would start a new block 3, 1, 2 and 2 so there are no additional multiples of 4.

29. 66

First give each brother 5 of the dollar bills leaving 10 of the bills left to share among the 3 brothers. Let's line up the 10 bills with 2 dividers like below :



We now need to arrange these in some way where the amount of bills in front of the first divider is how much the 1st brother gets, the amount between the dividers is what the 2nd brother gets and the amount after the 2nd divider is what the last brother gets. All we need to do then is find how many ways to arrange these 12 items. Any 12 items can be arranged in 12! ways. However, 10 of these items are identical, so we'd have to divide that 12! by 10! to account for all the ways the 10 bills could be arranged but that would be indistinguishable. We'd also have to divide by 2! to account for the ways the 2 dividers could be arranged that are also indistinguishable. So the number of ways to share the 10 bills (since 5 each were already given) is $\frac{12!}{10! \times 2!} = \frac{12 \times 11}{2 \times 1} = 66$.

Another way to look at this would be to say that the 10 bills and 2 dividers are twelve objects that need to be arranged in 12 spaces. We just need to choose which 2 spaces to place the dividers and the bills will fill in the rest of the spaces. That can be done in $\binom{12}{2} = \frac{12!}{2!(12-2)!} = \frac{12 \cdot 11}{2 \cdot 1} = 66$ ways.

30. $14\frac{2}{7}$

A decrease of x % can be shown algebraically by multiplying by $(1-x)$ where x is now the decimal equivalent (or fraction equivalent) of x %. Similarly an increase of x % can be shown by multiplying by $(1+x)$. So the following equation can help us find the value of x :

$$\begin{aligned} 60(1+x) &= 80(1-x) \\ 60+60x &= 80-80x \\ 140x &= 20 \\ x &= \frac{1}{7} \end{aligned}$$

Since we defined x as the decimal (fraction) equivalent of the percent we are looking for, we now need to multiply by 100 to get the percent we wanted : $\frac{1}{7} \times 100 = \frac{100}{7} = 14\frac{2}{7}$%.

1. 0.5

$33 \times 8 = 264$ more minutes are needed to write problems.
$106 \times 6 = 636$ more minutes are needed to write solutions.
$264 + 636 = 900$ more minutes are needed to finish the book.
$900 \div 30 = 30$ minutes per day are needed to finish the book.
30 minutes equals .5 hours still needed per day to finish the book.

2. 63

If d represents the distance traveled Saturday and t represents the time travelled on Saturday, then we can use the equation for distance, rate and time to describe his trip on Saturday as $d = 36t$. Since Sunday's trip is 24 miles longer and 18 minutes $\left(\frac{3}{10}\text{ hours}\right)$ shorter, the equation $d + 24 = 60\left(t - \frac{3}{10}\right)$ describes Sunday's trip. Replacing d in Sunday's equation with the $36t$ from Saturday's equation, we get :

$$36t + 24 = 60\left(t - \frac{3}{10}\right)$$
$$36t + 24 = 60t - 18$$
$$42 = 24t$$
$$\frac{7}{4} = t$$

Plugging $\frac{7}{4}$ in for t in Saturday's equation gives us : $d = 36 \times \frac{7}{4} = 63$ miles.

3. 3

If 30% of students got #13 wrong, then 15 students got it wrong. Each of those 15 students were assigned 4 questions for a total of 60 homework problems. If 95% of those questions were answered correctly then 5% were answered incorrectly. 5% of 60 questions is 3 questions answered incorrectly.

4. 32

If the hundreds digit is anything from 2 through 7, then there are 2 choices for the tens digit and 2 choices for the units digit. That's a total of $6 \times 2 \times 2 = 24$ numbers that satisfy the conditions. If the hundreds digit is 9, 8 or 1 there are some restrictions depending on cases so it is just easier to list out those numbers as follows :
989, 987, 898, 878, 876, 123, 121, 101
That's 8 additional numbers for a total of $24 + 8 = 32$ numbers.

5. 12

The hardest part of this problem is going to be deciding which coin to use for the variable on which all other coins will be based. After thinking about how the coin amounts are related to one another using d for dimes as our variable may be easiest, although there are other ways to start. Making a chart of the number of each coin as well as each coin's value can help keep all the information organized as follows :

Coin	Pennies	Nickels	Dimes	Quarters
# of the coin	$\frac{3}{2}d+3$	$\frac{3}{2}d$	d	$d+3$
Value of coin(cents)	1	5	10	25
Total value per type	$1\left(\frac{3}{2}d+3\right)$	$5\left(\frac{3}{2}d\right)$	$10d$	$25(d+3)$

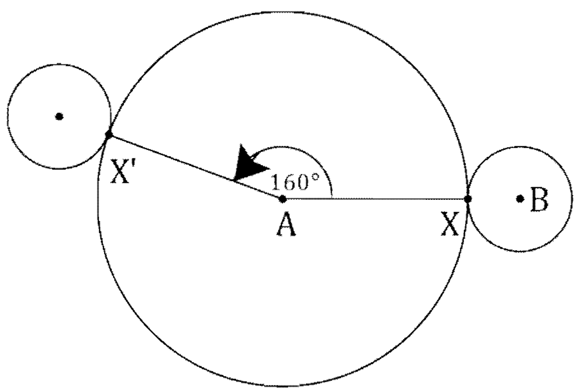
Now we can write an equation to give us the number of dimes.

$$1\left(\frac{3}{2}d+3\right)+5\left(\frac{3}{2}d\right)+10d+25(d+3)=430$$
$$\frac{3}{2}d+3+\frac{15}{2}d+10d+25d+75=430$$
$$44d+78=430$$
$$44d=352$$
$$d=8$$

If the number of dimes is 8, then the number of nickels is $\frac{3}{2}\times 8=12$.

6. 160

Circle B has a diameter of 4, so its circumference is 4π. If circle B makes 20 full revolutions it will have traveled 80π. Circle A has a diameter of 18 and a circumference of 18π. When circle B has traveled 72π it will have made four full trips around circle A. Circle B will still need to travel an additional 8π around circle A which is $\frac{8\pi}{18\pi}=\frac{4}{9}$ of the way around the circle. Four-ninths of a trip around a circle would mean that angle XAX' would form a $\frac{4}{9}\times 360=160°$ angle.



7. 13

For this problem we can write two equations each using two variables. Using m to represent the cost of a bucket of caramel popcorn, and d to represent the cost of a bucket of cheddar flavored popcorn we can write the two equations below :

$$6m + 9d = 93$$
$$9m + 7d = 107$$

Multiplying the first equation by 3 and the second equation by 2 gives us :

$$18m + 27d = 279$$
$$18m + 14d = 214$$

Subtracting the second equation from the first gives us :

$$13d = 65$$
$$d = 5$$

So the cost of a bucket of cheddar popcorn is \$5. Plugging that into the original first equation we get $6m + 9(5) = 93$ which means that $m = 8$ and a bucket of caramel popcorn costs \$8. So mark would pay \$5 + \$8 = \$13 for one bucket of each type.

8. 53

Using a for the number of adults and c for the number of children, we can write this one equation with both variables : $37a + 18c = 1733$. One equation with two variables would have an infinite number of ordered pair solutions, but since we know both a and c have to be positive integers that limits the number of solutions as well as gives us a technique for finding them. Since we only want integer solutions, we can use modular arithmetic to find integral ordered pair solutions.

$$37a + 18c \equiv 1733\ (mod\ 18)$$
$$36a + 1a + 18c \equiv 1728 + 5 (mod\ 18)$$
$$0 + 1a + 0 \equiv 0 + 5\ (mod\ 18)$$
$$a \equiv 5\ (mod\ 18)$$

Working in "mod 18" means that we are only looking at the remainders when values are divided by 18. Since $36a$, $18c$ and 1728 are all multiples of 18, they all have remainders of 0 when divided by 18. The modular congruence $a \equiv 5\ (mod\ 18)$ means that a is 5 more than a multiple of 18, so a could be 5, 23, 41, 59, ... Plugging in 5 for a in our original equation gives us $37(5) + 18c = 1733$ which means that $c = 86$. So if there were 5 adults at the reunion there would have to be 86 children, but we know there were more adults so that can't be right. If we add 18 adults the cost would go up 37(18) dollars. We could balance that out by lowering the number of children by 37. That gives us 23 adults and 49 children, but that still gives us more children than adults, so repeat that step to get $23 + 18 = 41$ adults and $49 - 37 = 12$ children. That fits the information that there were more adults, so there must have been $41 + 12 = 53$ people at the reunion. Check your answer by finding the cost for this answer :

$$37(41) + 18(12) = 1733$$

*See next page for a graphing calculator solution to this problem.

You could also take the original equation and solve for a : $a = \frac{1733-18c}{37}$. Using a graphing calculator enter the equation as $y_1 = \frac{1733-18x}{37}$. Go to Table Set and start the table at 1 and set the change in table (Δ table) to 1. Then look at the values in the table and scroll down for (x, y) pairs which are both positive integers. Remember that the y values represent the adults in this equation and we want more adults than kids so we want a pair where $y > x$.

1. 120

The triangular numbers are the sums of the consecutive counting numbers and they start like this :

1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, ...

Looking at the remainders when these numbers are divided by 4 we get :

1, 3, 2, 2, 3, 1, 0, 0, 1, 3, 2, 2, 3, 1, 0, 0, ...

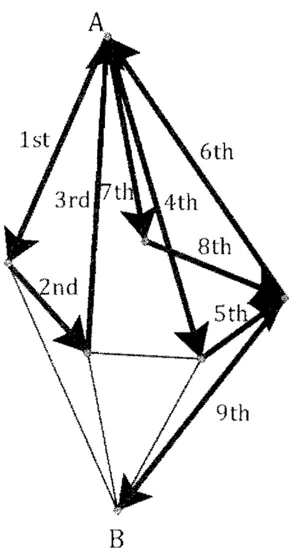
A clear pattern has emerged with a repeating block of 8 numbers. The sum of the remainders in those 8 is $1 + 3 + 2 + 2 + 3 + 1 + 0 + 0 = 12$. The pattern would repeat 10 times in the first 80 triangular numbers, so the sum of all the remainders would be $10(12) = 120$.

2. 80

These three-digit integers must use only the digits 1, 3, 5, 7 and 9. There are 5 choices for the hundreds digit. Since the tens digit cannot be the same as the hundreds digit, there are 4 choices for the tens digit. The units digit cannot be the same as the tens digit, but it can be the same as the hundreds digit so there are again 4 choices for the units digit. That's a total of $5 \times 4 \times 4 = 80$ three-digit integers with all odd digits and no two digits the same.

3. 27

The diagram below shows one way you could travel 9 of the 15 edges of the decahedron. Since each edge is 3 units long, the longest path is 27 units.



4. $\underline{\ 11\ }$

This is what has happened so far if the area of the original picture was :

$$A \cdot \frac{5}{4} \cdot \frac{3}{4} \cdot \frac{23}{20} = \frac{69}{64}A$$

We want the area to be 20% greater than A or $\frac{6}{5}A$. We can use the equation below to find out how much we need to alter the picture by :

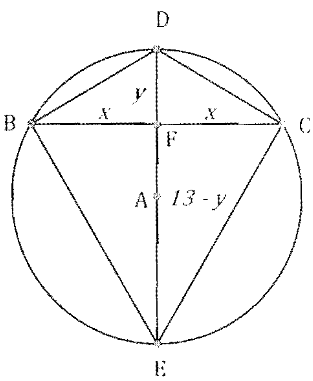
$$\frac{69}{64}A \cdot y = \frac{6}{5}A$$
$$y = \frac{6}{5} \cdot \frac{64}{69} = \frac{384}{345} \approx 1.113$$

Since y is the number we'd have to multiply by to change the area, the percent change would be $y - 1$. So the percent change is $1.113 - 1 = .113 = 11.3\% \approx 11\%$.

5. $\underline{\ 78\ }$

The diagram below shows the information given in the problem. When a diameter is perpendicular to a chord of a circle, then the chord is bisected (cut into 2 equal lengths). The two pieces have been marked x in the diagram. DF is marked y leaving the rest of the diameter, EF, to be marked $13 - y$.

Whenever two chords intersect in a circle, the products of the pieces of each chord are equal to each other as expressed in this equation: $x \cdot x = y(13 - y)$. This is called "Power of a Point". Normally a 2 variable equation with both variables being squared would not be very helpful, but since we know x and y are both integers we can just look for values of y in which $y(13 - y)$ is a perfect square so that x will be an integer. We know that $\overline{DF}$ is shorter than the radius, so DF < 6.5 units.



y	$y(13-y) = x^2$	x
1	$1(12) = 12$	$\sqrt{12}$
2	$2(11) = 22$	$\sqrt{22}$
3	$3(10) = 30$	$\sqrt{30}$
4	$4(9) = 36$	6
5	$5(8) = 40$	$\sqrt{40}$
6	$6(7) = 42$	$\sqrt{42}$

Now that we know $y = 4$ and $x = 6$, we need to find the area of quadrilateral BDCE. Since the quadrilateral has perpendicular diagonals, the area can be found by the formula $A = \frac{1}{2}(d_1 d_2)$ where d_1 and d_2 are the diagonals. So the area of BDCE is $\frac{1}{2}(12 \cdot 13) = 78$.

6. $\frac{7}{12}$

If the ratio of red to non-red jelly beans in Dylan's jar was 7 : 13, then $\frac{7}{20}$ of the jar was red and $\frac{13}{20}$ of the jar was non-red. Since 45 beans were added to the red and taken from the non-red the new ratio of red to non-red could be written as shown below where t is the total number of jelly beans in Dylan's jar at the start. We know that new ratio is 3 : 2, so we can write this equation.

$$\frac{\frac{7}{20}t + 45}{\frac{13}{20}t - 45} = \frac{3}{2}$$

Solving this equation we get:

$$2\left(\frac{7}{20}t + 45\right) = 3\left(\frac{13}{20}t - 45\right)$$
$$\frac{14}{20}t + 90 = \frac{39}{20}t - 135$$
$$225 = \frac{25}{20}t$$
$$t = 180$$

So there were 180 total jelly beans in Dylan's jar, $\frac{7}{20}$ of which makes 63 red jelly beans. If Brian added 45 red, then the fraction of the jelly beans that are red now that were red to start would be :

$$\frac{63}{63 + 45} = \frac{63}{108} = \frac{7}{12}$$

7. $\frac{5}{28}$

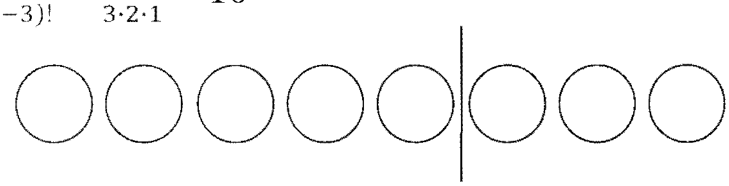
The 8 circles below represent the 8 plastic eggs lined up in the order that Fanny decides to open them. The vertical line is there to show that the three chocolate eggs must be to the left of the line so that they will all be within the first 5 eggs that will be opened. There are 8 eggs and 3 were chosen to have chocolate eggs in them which can be done in 56 ways as shown below :

$${}_8C_3 = \binom{8}{3} = \frac{8!}{3!\cdot(8-3)!} = \frac{8\cdot7\cdot6}{3\cdot2\cdot1} = 56$$

Similarly the number of ways to choose 3 of the first 5 eggs to place the chocolates in is :

$${}_5C_3 = \binom{5}{3} = \frac{5!}{3!\cdot(5-3)!} = \frac{5\cdot4\cdot3}{3\cdot2\cdot1} = 10$$

So the probability would be $\frac{10}{56} = \frac{5}{28}$.



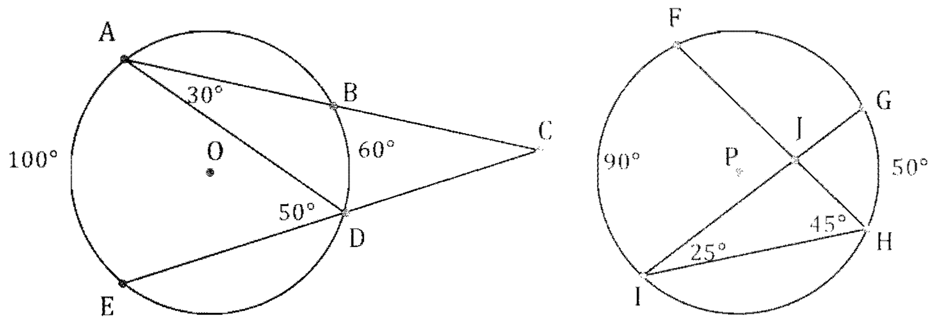
8. 20 & 70

In circle O, draw in chord AD. Inscribed angle ADE is half of minor arc AE, so the measure of angle ADE is 50°. Similarly the measure of angle CAD is 30° as it is half of minor arc BD. Angle ADC is supplementary to ADE, so the measure of angle ADC is 130°. The sum of the measures of angles CAD, ADC, and ACE is 180° since they are the three angles of a triangle, so the measure of ACE is 20°.

*note, use this technique with variable measure for the two minor arcs to prove to yourself that the measure of angle ACE is always half the difference of the 2 minor arcs.

In circle P, draw in chord IH. Inscribed angle FHI is half of minor arc FI, so the measure of angle FHI is 45°. By the same reasoning, the measure of angle GIH is 25°. Since angles GIH, FHI and IJH are all in the same triangle their sum is 180°. That means the measure of angle IJH is 110° and the measure of angle GJH is 70° since they are supplementary.

*note, use this technique with variable measure for the two minor arcs to prove to yourself that the measure of angle GJH is always the average of the 2 minor arcs.



9. −50

The sequence "A" is a geometric sequence with a common ratio of $\frac{2}{3}$. The sequence "B" is a geometric sequence with a common ratio of $\frac{4}{3}$. The n^{th} term of the sequence "A" can be found by $108 \times \left(\frac{2}{3}\right)^{n-1}$ while the n^{th} term of the sequence "B" can be found by $540 \times \left(\frac{4}{3}\right)^{n-1}$. So the 45^{th} term of A would be $108 \times \left(\frac{2}{3}\right)^{44}$ and the 48^{th} term of B can be found by $540 \times \left(\frac{4}{3}\right)^{47}$.

The ratio of those terms would then be :

$$\frac{108 \times \left(\frac{2}{3}\right)^{44}}{540 \times \left(\frac{4}{3}\right)^{47}} = \frac{1 \times \frac{2^{44}}{3^{44}}}{5 \times \frac{2^{94}}{3^{47}}} = \frac{1 \times 2^{44} \times 3^{47}}{5 \times 2^{94} \times 3^{44}} = \frac{1 \times 3^{3}}{5 \times 2^{50}} = \frac{27}{5} \times 2^{-50}$$

So when written in the form $\frac{27}{5} \times 2^{n}$, $n = -50$.

10. 182

To solve this problem we are going to need 4 variables and 4 equations. The variables will represent the cost of each color. We'll just use the first letter of each color for the variable for each. Se we have these 4 equations :

Equation #1 $3r + 2b + 4g + 1y = 39$
Equation #2 $2r + 4b + 3g + 1y = 36$
Equation #3 $5r + 2b + 1g + 3y = 41$
Equation #4 $4r + 3b + 2g + 1y = 39$

Combining equations 2 at a time to eliminate the variable y we get :

Equation #5 : EQ1 − EQ2: $1r - 2b + 1g = 3$
Equation #6 : 3 · EQ2 − EQ3 : $1r + 10b + 8g = 67$
Equation #7 : EQ4 − EQ2 : $2r - 1b - 1g = 3$

Combining equations 2 at a time again to eliminate the variable g we get :

Equation #8 : EQ5 + EQ7: $3r - 3b = 6$
Equation #9 : EQ6 + 8 · EQ7 : $17r + 2b = 91$

Combining those two equations we solve for r :

Equation #10 : 2·EQ8+3·EQ9 : $57r = 285$ so $r = 5$.

Plugging 5 in for r in equation #8 tells us that $b = 3$.
Plugging 5 in for r and 3 in for b into equation #7 tells us that $g = 4$.
Plugging 5 in for r, 3 in for b and 4 in for g into equation #1 tells us that $y = 2$.

Since Chaz bought 13 of each type, we can find the total cost of 1 of each type and then multiply by 13. So Chaz's total cost would be 13(5+3+4+2)=13(14) = 182 cents.