1. If $5005 = 7 \cdot 11 \cdot a$, then $a = 5005 \div 7 \div 11 = 65$. It is also helpful to know that $1001 = 7 \cdot 11 \cdot 13$, which means that $5005 = 5 \cdot 7 \cdot 11 \cdot 13$ which helps in determining that $a = 5 \cdot 13 = 65$ without having to do the long division. 1. 65

2. \$156 total divided by \$12 for each driveway equals 13 driveways. 2. 13

3. Hao read 35% of his 80 pages. That's $.35 \times 80 = 28$ pages. Jonathan read $.6 \times 80 = 48$. So Jonathan read $48 - 28 =$ 20 more pages than Hao. 3. 20

4. $24 \div 2 \times 3 + 5^2$
$24 \div 2 \times 3 + 25$
$12 \times 3 + 25$
$36 + 25$
61
4. 61

5. $30^2 = 900$, $31^2 = 961$, $32^2 = 1024$ 5. 32

6. Peijin has already gotten 8 questions wrong. If he ends up with 75% correct, then 25% of the questions would be wrong. If the 8 wrong questions make up the 25% that are wrong, then $3 \times 8 = 24$ questions make up the $25\% \times 3 = 75\%$ that are correct. Thus, there must be at least $24 + 8 = 32$ total questions. 6. 32

7. $600 = 2^3 \cdot 3^1 \cdot 5^2$. The number of positive integer factors can be found by adding 1 to each of the exponents and finding the product of the those sums. $(3 + 1)(1 + 1)(2 + 1) = (4)(2)(3) = 24$ factors. The reason you add 1 is that any factor of 600 will be divisible by some combination of the powers of the prime factors in the prime factorization of 600 from the 0 power up to the power in the prime factorization. So if the power in the prime factorization is 3, there are 4 possibilities for the power in any factor (0, 1, 2, 3). The 0 power always causes the need to add one to the exponents. 7. 24

8. $(6 \circledcirc 3) \circledcirc (4 \circledcirc 2)$
$\left(6 \cdot 3 + \frac{6}{3}\right) \circledcirc \left(4 \cdot 2 + \frac{4}{2}\right)$
$(18 + 2) \circledcirc (8 + 2)$
$(20) \circledcirc (10)$
$20 \cdot 10 + \frac{20}{10}$
$200 + 2$
202

8. 202

9. $5\% = \frac{1}{20}$ of the recommended daily allowance. If 16 grams is $\frac{1}{20}$ of the recommended daily allowance, then the recommended daily allowance is $20 \times 16 = 320$ grams.

9. 320

10. The arithmetic mean is the average. The average of 12 numbers is found by dividing the sum of the numbers by 12. If the average of the 12 numbers is $37\frac{2}{3}$, then the sum of the 12 numbers is $12 \times 37\frac{2}{3} = \frac{12}{1} \times \frac{113}{3} = \frac{1356}{3} = 452$. If 4 of the numbers are increased by 7, the total would be increased by $4 \times 7 = 28$. The new total would be $452 + 28 = 480$. Thus, the new average would be $480 \div 12 = 40$.

10. 40

11. If there are 560 shoes then there are $560 \div 2 = 280$ kids. If there are 4 boys for every 3 girls, then $\frac{3}{7}$ of the kids are girls. So the number of girls is $\frac{3}{7} \times 280 = 3 \times 40 = 120$.

11. 120

12. There were a total of $50 + 27 + 30 + 14 = 121$ entries in the raffle. Ivy and Andrew accounted for $50 + 27 = 77$ of the entries. The probability of Andrew or Ivy winning is $\frac{77}{121} = \frac{7}{11}$.

12. $\frac{7}{11}$

13. If a two-digit integer has x as the tens digit, and y as a units digit the two-digit integer could be represented algebraically as $10x + y$. If the digits were reversed, the integer would be $10y + x$. If the new integer is 36 greater, then:
$10x + y + 36 = 10y + x$
$9x + 36 = 9y$
$x + 4 = y$
That means the units digit is 4 more than the tens digit. The numbers that satisfy that rule are 15, 26, 37, 48, 59.

13. 59

14. For a positive integer to have an odd number of factors it must be a perfect square. The 20th smallest positive integer with an odd number of factors then is $20^2 = 400$.

14. 400

15. If the three-digit integer was *abc*, where *abc* does not mean *a* times *b* times *c*, but a three-digit integer, then the 6-digit number would be *abc*, *abc* which is $abc \times 1001$. So no matter what *abc* is, the 6-digit integer will always be divisible by 1001. Since $1001 = 7 \cdot 11 \cdot 13$, 13 is the largest prime factor that must be a factor of the 6-digit integer.

15. 13

16. 30% of 120 people = 36 people have no piercings
15% of 120 people = 18 people have 1 piercing
25% of 120 people = 30 people have 2 piercings
20% of 120 people = 24 people have 4 piercings
10% of 120 people = 12 people have 6 piercings
That's a total of $36(0) + 18(1) + 30(2) + 24(4) + 12(6) = 0 + 18 + 60 + 96 + 72 = 246$ piercings.

16. 246

17. While there are 8 points, there are only 7 units of space between those points. The distance between $\frac{11}{12}$ and $\frac{3}{8}$ is $\frac{11}{12} - \frac{3}{8} = \frac{22}{24} - \frac{9}{24} = \frac{13}{24}$. The point E is $\frac{4}{7}$ of that distance from A, which is $\frac{4}{7} \cdot \frac{13}{24} = \frac{13}{42}$. Adding $\frac{13}{42}$ to $\frac{3}{8}$, we find that the location of point E is $\frac{13}{42} + \frac{3}{8} = \frac{52}{168} + \frac{63}{168} = \frac{115}{168}$.

17. $\frac{115}{168}$

18. $216 = 2^3 \cdot 3^3$ and $72 = 2^3 \cdot 3^2$. If 216 is the LCM of 72 and N, N must have 3^3 in its prime factorization to force $3^3 = 27$ to be a factor of the LCM. So $N = 27 \cdot n$ where n is either $2^0, 2^1, 2^2$ or 2^3. So N = 27, 54, 108 or 216.

18. 4

19. There are 6 ways the wires could be hooked up: YRW, YWR, WRY, WYR, RWY and RYW. Only one of those can be correct, so the probability is $\frac{1}{6}$.

19. $\frac{1}{6}$

20. The expression $2(3x + 9)^2$ will never be negative. Since $2(3x + 9)^2$ is being subtracted from 12, the greatest this expression can ever be is $12 - 0 = 12$. In order for that to be the case $2(3x + 9)^2 = 0$ which means that $3x + 9 = 0$ and $x = -3$.

20. – 3

21. The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$. If a sphere has a radius half as big as the original sphere $\left(\frac{1}{2}r\right)$ then the smaller sphere's volume would be:

$V_{smaller} = \frac{4}{3}\pi\left(\frac{1}{2}r\right)^3 = \frac{4}{3}\pi\left(\frac{1}{8}r^3\right) = \frac{1}{8}\left[\frac{4}{3}\pi r^3\right]$.

The smaller sphere has $\frac{1}{8}$ of the volume of the larger, so when the smaller sphere is hollowed out, $\frac{7}{8}$ of the larger sphere will remain.

21. $\frac{7}{8}$

22. $\frac{4}{5} = \frac{248}{x}$

$4 \cdot x = 5 \cdot 248$

$4x = 1240$

$x = 310$

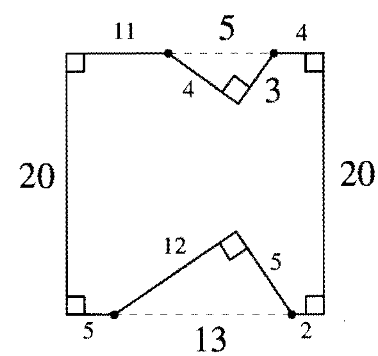
22. 310

23. The formula for circumference is $C = \pi \cdot d$. The larger circle has circumference 24π cm so the diameter is 24 cm. If the diameter is 24 cm, then the radius is 12cm. If the radius is 12 cm, then the area is $\pi \cdot 12^2 = 144\pi$ cm². The smaller circle has an area which is $44\frac{4}{9}$% as big. The area of the smaller circle is $\frac{4}{9} \cdot 144\pi = 64\pi$ cm². If the area is 64π cm², then the radius of the smaller circle is 8 cm, the diameter is 16 cm and the circumference is 16π cm.

23. 16π

24. First, draw in the missing parts of the sides of the square. The triangle on the bottom of the square has a hypotenuse of 13 inches. This can found by using the Pythagorean Theorem $(5^2 + 12^2 = 13^2)$. Knowing the 13 makes the side of the square equal to 20 inches. The hypotenuse of the triangle at the top of the square must be $20 - (11 + 4) = 5$ inches. Knowing the hypotenuse is 5 means the Pythagorean Theorem can again be used to find that the missing side of the triangle is 3 since $3^2 + 4^2 = 5^2$. The area of the whole square is $20^2 = 400$. The area of the top triangle is $\frac{3 \cdot 4}{2} = 6$ sq. in. and the area of the bottom triangle is $\frac{5 \cdot 12}{2} = 30$ sq. in. The area of the figure is $400 - 6 - 30 = 364$ sq. inches.

24. 364



25.

$$\begin{aligned} 25\% \text{ of } \$4 &= \$1.00 \\ 35\% \text{ of } \$2 &= \$0.70 \\ +\ 40\% \text{ of } \$3 &= \$1.20 \\ \hline 100\% &= \$2.90 \end{aligned}$$

A second, less abstract, way to do this problem would be to look at what would happen over a certain period of time that we can evenly take 25% and 35% of; let's say a 20-day period. Katie would get \$4 on 5 of the days (25% of 20). She would get \$2 on 7 of the days (35% of 20). She would get \$3 on 8 of the days (40% of 20). Katie would get a total of $\$4(5) + \$2(7) + \$3(8) = \$20 + \$14 + \$24 = \$58$ over 20 days for an average of $\frac{\$58}{20} = \$2.90$.

25. 2.90

26. If one of the angles of a triangle is 70°, the other two must have a sum of 110°. If the triangle is isosceles, it must have two angles which are equal. If another of the angles is also 70°, then the third angle must be 40°. If the 70° angle is not one of the two equal angles, then the other two must be equal and they would each be 55°. The possible values of M would then be 70 – 40 = 30 and 70 – 55 = 15. The sum of those values is 30 + 15 = 45.

26. 45

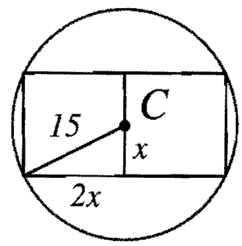
27. $xy - x - y = 23$
$xy - x - y + 1 = 24$
$(x - 1)(y - 1) = 24$
The product of $(x - 1)$ and $(y - 1)$ must be 24, so we need to list how many ordered pairs of numbers have a product of 24.
(1, 24); (2,12); (3,8); (4,6); (6,4); (8,3); (12,2); (24,1)
To know the values of x and y each number would need to have 1 added to it, but since we don't need to know the values of x and y, only how many pairs of (x, y) there are, the answer is 8 pairs.

27. 8

28. Draw a radius of the circle to one of the points where the circle touches the square. This newly draw radius completes a right triangle where one side of the triangle is half the length of the square's side (call that x) and another side of the triangle is the length of the side of the square ($2x$). The Pythagorean Theorem is used to find the value of x.

28. $225\pi - 360$

$x^2 + (2x)^2 = 15^2$
$x^2 + 4x^2 = 225$
$5x^2 = 225$
$x^2 = 45$
$x = 3\sqrt{5}$
$2x = 6\sqrt{5}$
$(2x)^2 = \left(6\sqrt{5}\right)^2$
$4x^2 = 180 =$ Area of a square



The area of the circle is $\pi(15^2) = 225\pi$ and the area of the region in the circle but outside the squares is $225\pi - 2(180) = 225\pi - 360$ sq. units.

29. If Allison's ride had been 28 miles to school, then it would have taken 2 hours to get to school and 7 hours to get home. It makes sense to choose 28 miles since it is the LCM of 14 and 4. The round trip would be 56 miles travelled in 9 hours. The average speed would be $56 \div 9 = 6\frac{2}{9}$. That is the average no matter how far the trip actually is.

29. $6\frac{2}{9}$

30. When 5 coins are tossed, the distribution of the number of heads and tails follows the 5th row of Pascal's triangle. There is 1 way to get 0 heads, 5 ways to get 1 heads, 10 ways to get 2 heads, 10 ways to get 3 heads, 5 ways to get 4 heads, and 1 way to get all 5 heads. Thirty-one of those have at least one head, and of those 26 have more than 1 head. So the probability of getting more than 1 head when you know there was at least 1 head is $\frac{26}{31}$.

30. $\frac{26}{31}$

1. From 309 to 341 → $\frac{341-309}{309} = \frac{32}{309} \approx .1035 = 10.3\%$ increase   1. 68.7

From 341 to 435 → $\frac{435-341}{341} = \frac{94}{341} \approx .2756 = 27.6\%$ increase

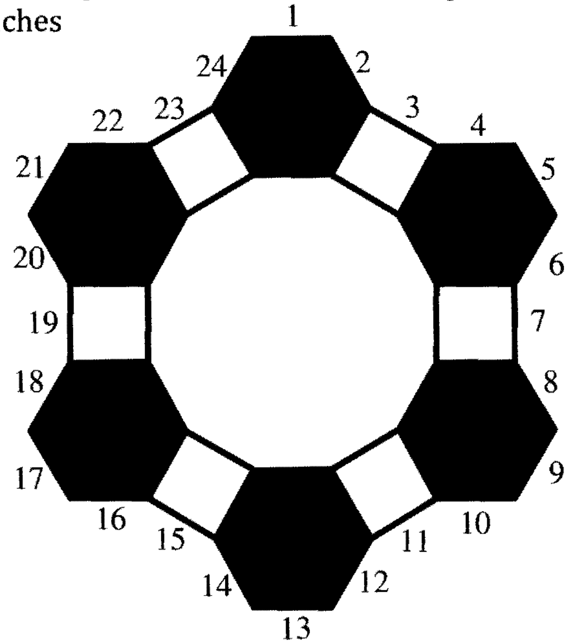
From 435 to 734 → $\frac{734-435}{435} = \frac{299}{435} \approx .6873 = 68.7\%$ increase

From 734 to 870 → $\frac{870-734}{734} = \frac{136}{734} \approx .1852 = 18.5\%$ increase

From 870 to 652 → $\frac{652-870}{870} = \frac{-218}{870} \approx -.2505 = 25.1\%$ decrease

From 652 to 759 → $\frac{759-652}{652} = \frac{107}{652} \approx .1641 = 16.4\%$ increase

2. If the dodecagon has a perimeter of 36 inches, then each side is $36 \div 12 = 3$ inches. The new shape will have 24 sides each 3 inches long, so the perimeter of the entire shape will be $24 \times 3 = 72$ inches   2. 72



3. If h is the cost of a hot dog and s is the cost of a soda, then these two equations represent the given information.   3. 10 or 10.00

$4h + 3s = 22.50$

$3h + 4s = 21.25$

Adding the two equations together results in the equation:

$7h + 7s = 43.75$

$h + s = 6.25$

$3h + 3s = 18.75$

If three of each cost \$18.75 then Philip's extra hot dog must have cost \$22.50 – \$18.75 = \$3.75. Yuankai bought 2 hot dogs and a soda which would cost \$6.25 (one of each) + 3.75 (extra hot dog) = \$10.00

4. In order for the sum of three prime numbers to be 100, one of them must be 2, because if all three primes were odd, the sum would be odd. That makes it easier to list the possible sets of three primes with a sum of 100. 4. 4514

$(2, 19, 79) \rightarrow 2 \cdot 19 \cdot 79 = 3002$

$(2, 31, 67) \rightarrow 2 \cdot 31 \cdot 67 = 4154$

$(2, 37, 61) \rightarrow 2 \cdot 37 \cdot 61 = 4514$ which is the greatest

5. The sum of the first N odd integers is always N^2. Thus, the sum of the 10 odd integers from 1 to 19 can be calculated as $10^2 = 100$. Therefore, the square of sum of the odd integers from 1 to 19 is $100^2 = 10{,}000$. The sum of the 10 even integers from 2 to 20 will be 10 more than the sum of the odds from 1 to 19, so the sum is 110. The square of 110 is $110^2 = 12{,}100$ which is $12{,}100 - 10{,}000 = 2{,}100$ larger. 5. 2100

6. The chart below shows how many problems are written each day and the total written at the end of each day. 6. 10

	day 1	day 2	day 3	day 4	day 5	day 6	day 7	day 8	day 9	day 10
day n	3	4	6	10	18	34	66	130	258	514
total	3	7	13	23	41	75	141	271	529	1043

After 9 days Mr. Frost had written 529 problems. On the 10th day Mr. Frost reached 1043 total problems so the 1000th problem was written on day 10.

7. If the side of the square is 14 inches, the diameter of each circle is 7 inches. The radius would be 3.5 inches. The area of each circle would be $\pi r^2 = \pi(3.5^2) = 12.25\pi$. The combined area of the 4 circles would be $4(12.25\pi) = 49\pi$. The area of the square is $14^2 = 196$. The shaded region's area is $196 - 49\pi = 196 - 49(3.1415) = 42.0665 \approx 42.07$ 7. 42.07

8. The table below lists the 32 ways to flip a coin 5 times. The cases in which heads did not come up twice are highlighted. There are 13 cases, so the probability is $\frac{13}{32}$. 8. $\frac{13}{32}$

HHHHH	HTHHH	HTHHT	THHTH	HHTTT	THTHT	TTHTH	TTHTT
HHHHT	THHHH	THHHT	HTTHH	HTHTT	TTHHT	TTTHH	TTTHT
HHHTH	HHHTT	HHTTH	THTHH	THHTT	HTTTH	HTTTT	TTTTH
HHTHH	HHTHT	HTHTH	TTHHH	HTTHT	THTTH	THTTT	TTTTT

1. $$\frac{40}{|n+1|} + 2 \geq 7$$
$$\frac{40}{|n+1|} \geq 5$$
$$40 \geq 5|n+1|$$
$$8 \geq |n+1|$$
$$8 \geq n+1 \geq -8$$
$$7 \geq n \geq -9$$

$n \in \{7, 6, 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, -6, -7, -8, -9\}$
That gives 17 values for n. However, n cannot equal -1 as it would make the denominator of the original fraction 0, which is not possible, so there are only 16 values for n.

1. 16

2. $$.07(s - 80{,}000) = 11{,}200$$
$$s - 80{,}000 = 160{,}000$$
$$s = 240{,}000$$

2. 240,000

3. $28\frac{4}{7}\% = \frac{2}{7}$, $27\frac{7}{9}\% = \frac{5}{18}$, and $28\% = \frac{7}{25}$
The number of kids competing in each grade must be the same, so the numerators of the fractions must be equal.
$\frac{2}{7} = \frac{70}{245}$
$\frac{5}{18} = \frac{70}{252}$
$\frac{7}{25} = \frac{70}{250}$
So there are 70 teams, but a total of $245 + 252 + 250 = 747$ kids at the Clarke School.

3. 747

4. If Beverly, Ender and Cantor must sit together, we can arrange their seating as if those three are just one person. So, initially, we pretend as if there are only 5 friends and there are $5! = 120$ ways to arrange them. Now we have to take into account that we counted Beverly, Ender and Cantor as one person. Those three friends can arrange themselves in $3! = 6$ ways. So for each of the 120 ways there are 6 ways to arrange the three friends that must sit together. That's $120 \cdot 6 = 720$ ways for the friends to sit in the row.

4. 720

5. If three primes have a sum equal to 67, none of the three primes can be even (none can be 2). Listing sets of primes in order from least to greatest makes counting them easier. 5. 15

(3, 5, 59)	(7, 13, 47)	(11, 13, 43)	(17, 19, 31)
(3, 11, 53)	(7, 17, 43)	(11, 19, 37)	
(3, 17, 47)	(7, 19, 41)		
(3, 23, 41)	(7, 23, 37)	(13, 17, 37)	
	(7, 29, 31)	(13, 23, 31)	
(5, 19, 43)			

That's a total of 15 distinct sets of three distinct primes.

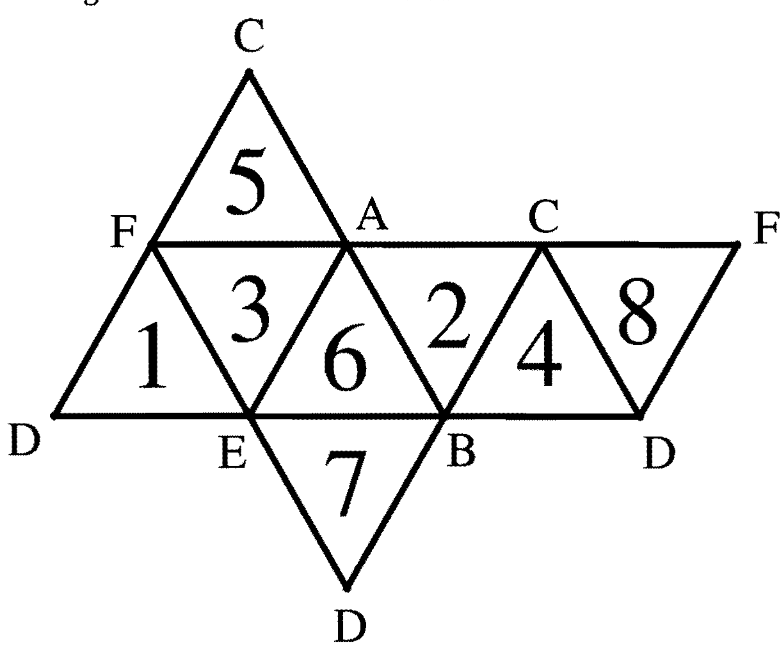
6. The 4th power of 11 is 14,641 while 100 squared is 10,000, therefore, there is no overlap between the two sets of numbers. Also, there are 100 numbers in each set and the units digits of all numbers are equally dispersed between the 10 possible units digits. When the units digits 0 through 9 are squared the new units digits are 0, 1, 4, 9, 6, 5, 6, 9, 4, 1. When the units digits 0 through 9 are raised to the 4th power, the new units digits are 0, 1, 6, 1, 6, 5, 6, 1, 6, 1. Of those twenty new possible units digits, 12 of the 20 are 1 or 6 which both have a remainder of 1 when divided by 5. That's $\frac{12}{20} = \frac{6}{10} = 60\%$. 6. 60

7. Dealing with the socks first, if ABC is the correct order for the three boys then BCA and CAB are the ways that none of the boys got the right socks. There are $3! = 6$ ways to distribute the socks, so the probability that none of the three boys got the right socks is $\frac{2}{6} = \frac{1}{3}$. Now let's deal with the shoes. There are ${}_5C_2 = \frac{5!}{(5-2)!2!} = 10$ ways for two of the five boys to get the right shoes. The other 3 boys have 2 ways to pick all the wrong shoes (as shown earlier). So there are $10 \cdot 2 = 20$ ways for 2 boys to get the right shoes and 3 boys to get the wrong shoes out of a possible $5! = 120$ ways to distribute the shoes. That's a probability of $\frac{20}{120} = \frac{1}{6}$. Finally, the probability of the socks and the shoes being distributed as asked is $\frac{1}{3} \cdot \frac{1}{6} = \frac{1}{18}$. 7. $\frac{1}{18}$

8. If there are 7 numbers and the median is 84, then 84 is the middle number. Because of the dual mode, there are either two 78s and two 88s, or three of each. If there were three of each the mean would not be 84 though, so there must be two of each. So we need to find possibilities for the last two numbers. A 7-number list with a mean of 84 would have a sum of $7(84) = 588$. The five numbers we know so far have a sum of 416. The remaining two numbers must have a sum of $588 - 416 = 172$. One of the numbers must be bigger than 84 and one must be smaller. Possible pairs would be:
83 & 89, 82 & 90, 81 & 91, 80 & 92, 79 & 93, 78 & 94,
77 & 95, 76 & 96, 75 & 97, 74 & 98, 73 & 99, and 72 & 100.
That's 12 possible pairs for the remaining numbers except that one of the pairs includes a 78 which would give the list three 78s and no longer a dual mode. So only 11 of the pairs would fit the 7-number list we are looking for.

8. 11

9. In the diagram below, the vertices have been labeled to show which other vertices they connect to. The sum of the faces around each vertex is calculated below.
$A = 5 + 3 + 6 + 2 = 16$
$B = 7 + 6 + 2 + 4 = 19$
$C = 2 + 4 + 8 + 5 = 19$
$D = 1 + 7 + 4 + 8 = 20$
$E = 1 + 3 + 6 + 7 = 17$
$F = 1 + 3 + 5 + 8 = 17$
So the greatest sum that can be obtained is 20.



9. 20

10. The chart below shows the results of the test for people who both do and do not have the disease. From the chart, it is clear that $.00049 + .01999 = .02048 = 2.048\%$ of people are told, "Yes" they do have the disease. However, only $.00049 = .049\%$ of people are told "Yes" and actually do have the disease. That means that only $\frac{.00049}{.02048} \approx .0239 \approx 2.4\%$ of people who are told they have the disease actually have it!

10. 2.4

X	Has Disease .0005	No Disease .9995
Test Accurate .98	Told Yes .00049	Told No .9751
Test Not Accurate .02	Told No .00001	Told Yes .01999