1. $\frac{12 - 2(2^3 - 5 \times 2^4)}{18 \div 3 \times 2} = \frac{12 - 2(8 - 5 \times 16)}{18 \div 3 \times 2} = \frac{12 - 2(8 - 80)}{18 \div 3 \times 2} =$

$\frac{12 - 2(-72)}{18 \div 3 \times 2} = \frac{12 - (-144)}{6 \times 2} = \frac{156}{12} = 13$

1. 13

2. Phil missed 10 out of 24 shots which equals:
$\frac{10}{24} = \frac{5}{12} = .41\bar{6} = 41\frac{2}{3}\%$

2. $41\frac{2}{3}$

3. The prime digits Stephan has to choose from are 2, 3, 5 and 7. He has 4 choices for the first digit in the code. Since he doesn't want to use the same digit twice in a row he only has 3 choices for the 2nd digit. For the 3rd digit the only digit he can't use is the digit chosen for the 2nd spot but he can use the 1st digit that was chosen, so he has 3 choices again. For the 4th digit he can use either of the digits from the 1st or 2nd choices, but not the digit that was the 3rd choice, so again 3 choices. Therefore, the total number of codes would be $4 \cdot 3 \cdot 3 \cdot 3 = 108$

3. 108

4. According to the chart 2 people got 5 questions correct, 4 people got 6 correct, 7 people got 7 correct, 8 people got 8 correct, 5 people got 9 correct, and 4 people got 10 correct. We can find the total number correct by finding $2 \cdot 5 + 4 \cdot 6 + 7 \cdot 7 + 8 \cdot 8 + 5 \cdot 9 + 4 \cdot 10$. Thus, the class got $10 + 24 + 49 + 64 + 45 + 40 = 232$ questions correct.

4. 232

5. If Dawon paid \$9.42 for 6 cups, then each cup costs \$9.42 ÷ 6 = \$1.57. Sean paid for his and Caron's coffee so he paid for 7 cups. To find Sean's total you multiply \$1.57 by 7 or just add \$1.57 to Dawon's total since Sean paid for one more cup than Dawon. \$1.57 × 7 = \$10.99 or \$9.42 + \$1.57 = \$10.99.

5. 10.99

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6. The average of Mikah's three throws was 48 mph; thus, the sum of the speeds of his three throws must have been $48 \cdot 3 = 144$. It is given that one of the throws was 44 mph, so the other two must have added up to $144 - 44 = 100$. If we know the sum of the other two than we just need to divide the sum by 2 to get the average. Therefore, the average speed of the other two throws was $100 \div 2 = 50$ mph. — 6. 50

7. First we simplify $\frac{83!}{81!}$ and $\frac{84!}{82!}$ to get $\frac{83!}{81!} = \frac{83 \cdot 82 \cdot 81!}{81!} = 83 \cdot 82$ and $\frac{84!}{82!} = \frac{84 \cdot 83 \cdot 82!}{82!} = 84 \cdot 83$. Since 82 and 84 have only 2 as a common factor, the greatest common factor of $83 \cdot 82$ and $84 \cdot 83$ is $83 \cdot 2 = 166$. — 7. 166

8. Since we are only looking for the units digit of the value of this power, we only really need to know the units digit of $7^{1234567}$. Looking at the first few powers of 7, we get: $7^1 = 7$, $7^2 = 49$, $7^3 = 343$, $7^4 = 2401$, $7^5 = 16807$. So far the units digits have formed the sequence: 7, 9, 3, 1, 7 and will continue as 7, 9, 3, 1, 7, 9, 3, 1... repeating every 4 powers. Obviously we don't want to list these units digits over and over again until we get to the 1,234,567th power. We know they repeat every 4th power, so we can say every power that is divisible by 4 will have the same units digit, namely 1, the units digit of 7^4. The closest multiple of 4 less than 1,234,567 is 1,234,564, so $1{,}234{,}567^{1{,}234{,}564}$ has a units digit of 1. $1{,}234{,}567^{1{,}234{,}565}$ would have a units digit of 7, $1{,}234{,}567^{1{,}234{,}566}$ would have a units digit of 9, and finally $1{,}234{,}567^{1{,}234{,}567}$ would have a units digit of 3. For future problems of this nature, the only real work necessary is to find the remainder of 1,234,567 when divided by 4. In this case, the remainder is 3, so $1{,}234{,}567^{1{,}234{,}567}$ ends in the same digit as $1{,}234{,}567^3$ which ends in the same digit as 7^3 which ends in 3. — 8. 3

9. The segment EF breaks rectangle ABCD into two smaller rectangles. Each of the two smaller rectangles has a shaded triangle in it. Each triangle has the same base (AD or BC) as the rectangle it is in as well as having the same height (DE or CE) as the rectangle is it in. That means each shaded triangle has half the area of the rectangle it is in also meaning that half the combined area is shaded. Therefore, 50% of ABCD is shaded. — 9. 50

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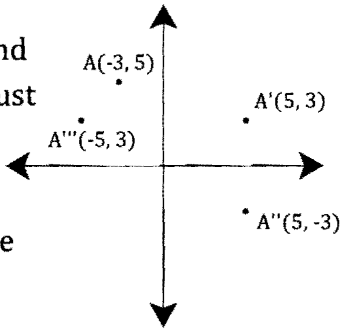
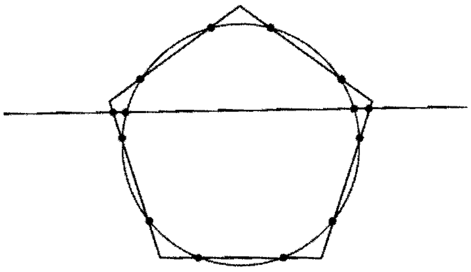
10. The diagram to the right shows the three transformations used in finding that the coordinates of A''' are $(-5, 3)$. The first rotation of 90° can be found using the idea that perpendicular lines have slopes that are the opposite of the reciprocal of each other. Since we are rotating around the origin, we can find the slope of the line through the origin and A as $-\frac{5}{3}$, so the slope of the line through the origin and A' must be $\frac{3}{5}$. That means the coordinates of A' are $(5, 3)$ since the distance from the origin must also be the same as it was before rotating. The next transformation simply changes the y-coordinate from 3 to -3 giving us A''$(5, -3)$. The last rotation of 180° around the origin simply multiplies both coordinates by -1 giving us A'''$(-5, 3)$.

10. (- 5, 3)



11. The diagram below shows how a line, a circle and a regular pentagon can have 14 points of intersection.

11. 14



12. The important thing to remember about triangle side lengths is that the sum of the lengths of any two of the sides is always greater than the length of the third side. So we are looking for combinations of three positive integers with a sum of 12 in which any two of the numbers have a sum greater than the third (so 1, 2, 9 would not be a valid set of side lengths for a triangle). The possible ways to do this are: (2, 5, 5); (3, 4, 5); (4, 4, 4). Any other set of three positive integers with a sum of 12 is just a reordering of one of those three, or has two of the numbers with a sum less than or equal to the third. Thus, there are only 3 distinct triangles with integer side lengths and a perimeter of 12 units.

12. 3

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13. When rolling two twenty-sided dice numbered from 1 to 20, you could achieve any sum from 2 through 40. However, the only sum you could achieve no matter what the result of the first role is would be 21. For each of the 20 ways to roll the first die there will still be 1 way to roll the next die to get a sum of 21. 13. 21

14. In order to find the average of a group of numbers you need to divide the sum of the numbers by how many numbers were summed. In this problem we know how many numbers are being added, but not what they are and not their sum. Let's call the sum "x". The average of the numbers then would be $\frac{x}{35}$. John mistakenly divided the sum by the average meaning he calculated this: $x \div \frac{x}{35} = \frac{x}{1} \cdot \frac{35}{x} = 35$. The incorrect steps provided 35 as the average which we are told turned out to be the correct average, so 35 was the average. 14. 35

15. $|x^2 + 78| \leq 144$ 15. -8

$-144 \leq x^2 + 78 \leq 144$

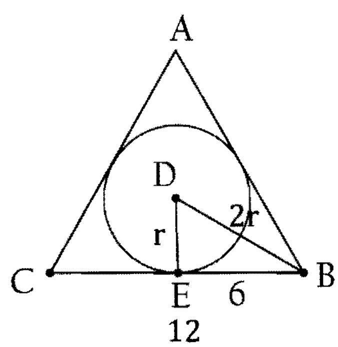
$-222 \leq x^2 \leq 66$ so $0 \leq x^2 \leq 66$ since x^2 is always positive.

$x \leq 8.124 \ldots$ and $x \geq -8.124 \ldots$

The least integral value of x that fits is -8.

16. Triangle ABC is an equilateral triangle, thus all of the angles of ABC measure 60°. The radius of the circle, when drawn to the point where the circle is tangent to $\overline{CB}$, is perpendicular to $\overline{CB}$. Thus, $\overline{DE}$ is perpendicular to $\overline{CB}$. $\overline{DB}$ would cut angle EBD in half (bisect) leaving angle BDE to be 60 degrees. Now we can see that triangle DEB is a 30-60-90 triangle, which means that $\overline{DB}$ is twice as long as $\overline{DE}$ and BE is equal to the square root of 3 times DE (which could be found using the Pythagorean Theorem). So $EB = \sqrt{3} \cdot r \rightarrow 6 = \sqrt{3}r \rightarrow r = \frac{6}{\sqrt{3}}$. Thus, the area of the circle is:

$A = \pi r^2 = \pi \left(\frac{6}{\sqrt{3}}\right)^2 = \pi \cdot \frac{36}{3} = 12\pi$. 16. 12π



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17. The prime factorization of 4200 is $2^3 \cdot 3^1 \cdot 5^2 \cdot 7^1$. Any factor of 4200 could be divisible by 2 anywhere from 0 to 3 times (4 choices), be divisible by 3 either 0 or 1 times (2 choices), be divisible by 5 anywhere from 0 to 2 times (3 choices), and be divisible 7 either 0 or 1 times (2 choices). That means there are a total of $4 \cdot 2 \cdot 3 \cdot 2 = 48$ factors. However, 1, 2, 3, 5 and 7 are not composite (1 is neither prime nor composite.), so there are $48 - 5 = 43$ composite factors.

17. 43

18. The least common multiple of 8 and 6 is 24, so let's start by looking at how much Sam will make if he buys 24 oranges. He would have to buy three bags totaling $3 \times 2.75 = \$8.25$. He would then sell those 24 oranges in four bags for a total of $4 \times 2.50 = \$10.00$. Sam would make $10.00 - 8.25 = 1.75$ when he buys three bags. Since $35 \div 1.75 = 20$ Sam would have to buy three bags of oranges 20 times. That's a total of 60 bags of oranges.

18. 60

19. $\frac{2}{x+3} = 5 \to 2 = 5x + 15 \to -13 = 5x \to x = \frac{-13}{5}$

$\frac{3}{x+2} = \frac{3}{\frac{-13}{5}+2} = \frac{3}{\frac{-3}{5}} = -5$

19. -5

20. $\frac{1}{1+\frac{2}{3+\frac{5}{8+13}}} = \frac{1}{1+\frac{2}{3+\frac{5}{21}}} = \frac{1}{1+\frac{2}{\frac{68}{21}}} = \frac{1}{1+\frac{21}{34}} = \frac{1}{\frac{55}{34}} = \frac{34}{55}$

20. $\frac{34}{55}$

21. An equation can help solve this one.
$x = .4x + 54 \to .6x = 54 \to x = 90$

21. 90

22. $7! = 2^4 \cdot 3^2 \cdot 5 \cdot 7$.
213! is divisible by two 208 times. The way you calculate that is by dividing 213 by 2 (to get 106 ignoring remainders), then dividing 106 by 2 (53) and continuing to repeat the division by 2. Then adding up all the quotients: $106 + 53 + 26 + 13 + 6 + 3 + 1 = 208$. Repeating that process for factors of 3, 5 and 7 gives us a partial prime factorization of $213! = 2^{208} \cdot 3^{103} \cdot 5^{51} \cdot 7^{34}$ ... Since 7! is divisible by 2^4, we have enough twos for 7! to "fit" into 213! $208 \div 4 = 52$ times. Similarly, since 7! is divisible by 3^2, we have enough threes for 7! to "fit" into 213! $103 \div 2 = 51$ times. Since 7! is divisible by 5 and 7 only once each, we can just look at the exponents of those factors in the prime factorization of 213!. That shows us that there are only enough sevens in 213! for 7! to "fit" into 213! 34 times.

22. 34

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23. Teesha has 60 ounces of 35% lemon juice which means her drink has $60 \times .35 = 21$ ounces of lemon juice. If Bill is adding water to her drink, his new drink will still have 21 ounces of lemon juice which we need to represent 15% of the drink. If Bill's drink has d ounces, the equation $.15d = 21 \rightarrow d = 140$ ounces. If Bill's drink would need 140 ounces then he needs to add $140 - 60 = 80$ ounces of water.

23. 80

24. We could line up the metal discs and then place two dividers among them to divide up the discs into three groups. There would be 7 choices for where to put the first divider. There would then be 8 places to place the second divider (the extra space is created by being able to place it on either side of the first divider). So, with 7 choices for the first divider and 8 for the second, there are $7 \cdot 8 = 56$ ways to place the dividers. Except that for every way you could place dividers 1 and then 2, they could have been placed in reverse (2 and then 1) without changing the way the discs were divided up so it is necessary to divide the total choices by 2. That means there are $56 \div 2 = 28$ ways to place the dividers and, therefore, distribute the discs. Another way to solve this problem is to add 2 extra discs giving a total of 8. Then choose two of the discs to be the dividers for the other 6. We count that total by calculating ${}_8C_2 = \frac{8!}{(8-2)! \cdot 2!} = \frac{8 \cdot 7}{2 \cdot 1} = \frac{56}{2} = 28$ ways to divide up the discs.

24. 28

25. The probability that he assigns homework on both days would be $\frac{5}{6} \cdot \frac{5}{7} = \frac{25}{42}$. The probability that he assigns homework neither day would be $\frac{1}{6} \cdot \frac{2}{7} = \frac{2}{42}$. The only other possibility is that he assigns homework on one day but not both. We can calculate that by:

$$1 - \left(\frac{25}{42} + \frac{2}{42}\right) = 1 - \frac{27}{42} = \frac{15}{42} = \frac{5}{14}$$

25. $\frac{5}{14}$

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26. First, let's call what Ike gets paid "x". If Ike gets 20% more than Spike that means he gets $\frac{6}{5}$ of Spike's pay, or that Spike gets $\frac{5}{6}$ of Ike's pay. Thus, Spike gets $\frac{5}{6}x$. Similarly, since Ike gets 30% less than Mike that means Ike gets $\frac{7}{10}$ of Mike's pay or Mike gets $\frac{10}{7}$ of Ike's pay or $\frac{10}{7}x$. Together, the three then make $\frac{5}{6}x + x + \frac{10}{7}x = \frac{137}{42}x$. If they make 1370 cents use the equation $x = 1370$ which yields Ike earning $x = 420$ cents or \$4.20

26. 4.20

27. The problem tells us that 80% or $\frac{4}{5}$ of the dogs were adopted and that 37.5% or $\frac{3}{8}$ of the cats were adopted. Since one cat was adopted for every one dog the numerators of those fractions must be the same. That gives us $\frac{12}{15}$ of the dogs and $\frac{12}{32}$ of the cats. That means that if 12 of each animal were adopted it would be out of a total of $15 + 32 = 47$ animals. So $\frac{12+12}{15+32} = \frac{24}{47}$ of the animals were adopted. The only fraction equivalent to $\frac{24}{47}$ with a denominator between 50 and 100 is $\frac{48}{94}$. So 48 animals were adopted, 24 of which would be cats. Using this proportion: $\frac{24}{x} = \frac{3}{8}$ we can find the total number of cats to be 64.

27. 64

28. Since the problem is asking for the remainder when the sum is divided by 10, it is really just asking for the units digit. This is similar to the question asked in problem number 8. Finding the units digit of 2^{2008} requires finding the units digit of the first few powers of 2 which are 2, 4, 8, 16, 32, 64... The units digit repeats after 4 powers, which is true for all numbers, although some have cycles of 1 or 2. Since 2008 is divisible by 4, 2^{2008} has the same units digit as 2^4. The same is true for 3^{2008}, 5^{2008} and 7^{2008}. $2^4 = 1\underline{6}, 3^4 = 8\underline{1}$, $5^4 = 62\underline{5}, 7^4 = 240\underline{1}$. The sum of the units digits of those 4 is the same as the sum of the units digits of the 2008th power of the same four bases. So the units digit of $2^{2008} + 3^{2008} + 5^{2008} + 7^{2008}$ is $6 + 1 + 5 + 1 = 1\underline{3}$.

28. 3

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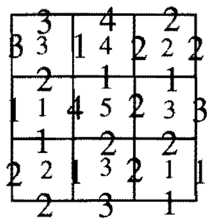
29. The surface area of the top and bottom of this arrangement will be 9 square inches no matter how many cubes are on each square of the grid. If one "tower" has 4 blocks and is next to a tower of 2 blocks, the 4 block tower would stick out 2 cubes higher than the two block tower so 2 square inches count towards the surface area. The second grid below has numbers between each tower that show how many squares are exposed on each side of each tower. The sum of those numbers is $3 + 4 + 2 + 3 + 1 + 2 + 2 + 2 + 1 + 1 + 1 + 4 + 2 + 3 + 1 + 2 + 2 + 2 + 1 + 2 + 1 + 2 + 3 + 1 = 48$ plus the 9 on top and 9 on bottom gives a total of 66. Thus, the surface area is 66 square inches.

29. 66

3	4	2
1	5	3
2	3	1



30. The two coaches start out 200 yards apart which equals 600 ft. Each second the coaches get 6 ft closer to each other, so it will take 100 seconds for them to reach each other. So Kim will run for 100 seconds at 20 feet per second for a total of 2000 feet.

30. 2000

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1. Mr. Cinema's TV has a ratio of 16 wide to 9 high and is 30 high. Solving the proportion $\frac{16}{9} = \frac{x}{30}$ gives the width of the TV as $53\frac{1}{3}$ inches. The viewing area of the DVD has a ratio of 2.35 wide to 1 high. Solving the proportion $\frac{53\frac{1}{3}}{x} = \frac{2.35}{1}$ gives the height of the viewing area of the latest DVD as $22.695 \approx 22.7$ inches.

1. 22.7

2. The prime factorization of 1000 is $2^3 \cdot 5^3$, so N is not 1000. $1001 = 7^1 \cdot 11^1 \cdot 13^1$, so N must be 1001. $a = 7, b = 11$ and $c = 13$ so $a + b + c = 7 + 11 + 13 = 31$.

2. 31

3. When the price of the TV is increased by 50% it becomes $\frac{3}{2}$ times as expensive as the previous price. When the price is decreased by 50% it becomes $\frac{1}{2}$ times as expensive. The result of the four price changes described in the problem can be described mathematically as $\frac{3}{2} \cdot \frac{1}{2} \cdot \frac{3}{2} \cdot \frac{1}{2} = \frac{9}{16}$ times as expensive as the original price. Solving the equation $\frac{9}{16}x = 450$, with x representing the original price, we find that the original price was \$800 or \$800.00.

3. 800 or 800.00

4. Writing N as $100a + 10b + c$ where a, b and c are the digits of N means that M can be written as $100c + 10b + a$. The difference of N and M would be N – M which is equal to $(100a + 10b + c) - (100c + 10b + a) = 99a - 99c = 99(a - c)$. That value will always be divisible by 99 no matter what a and c are, and b does not actually affect the difference.

4. 99

5. The prime factorization of 10,920 is $2^3 \cdot 3 \cdot 5 \cdot 7 \cdot 13$. In order for a number to be a perfect square, each of the exponents in the prime factorization must be even. In order for that to happen, 10,920 would need to be multiplied by $2 \cdot 3 \cdot 5 \cdot 7 \cdot 13 = 2730$.

5. 2730

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6. According to the problem, $x^2 - y^2 = 300$. Factoring the left side we get: $(x+y)(x-y) = 300$. The difference between $(x+y)$ and $(x-y)$ is $2y$, which is an even number. That means that $(x+y)$ and $(x-y)$ are either both even or both odd. Since their product is even, they can't both be odd. The only ways two positive even numbers can have a product of 300 are 150×2, 50×6 and 30×10. The ordered pairs (x, y) that would give the products are (76, 74), (28, 22) and (20, 10). The values of A would then be: $76 + 74 = 150$, $28 + 22 = 50$ and $20 + 10 = 30$. The sum of those values is $150 + 50 + 30 = 230$.

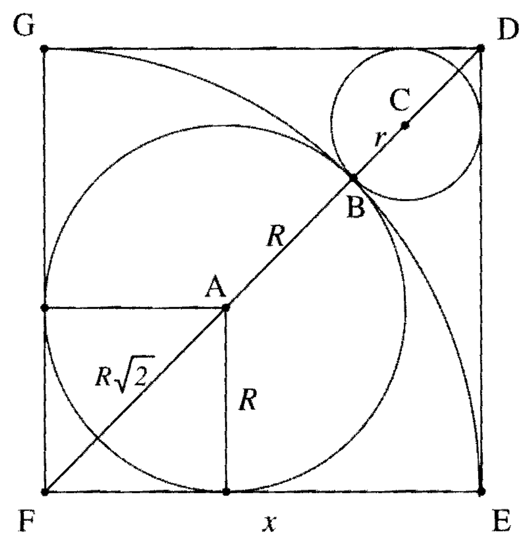
6. 230

7. Since b and c are each used twice, they should be the largest two values. Since b is multiplied by 5 and 3, b should be the largest of the values (13). Now substitute 13 for b to get:
$3ab + 5bc + 2cd = 39a + 65c + 2cd$
Since c is multiplied by 65 and $2d$, it should be the biggest remaining value (11). Substituting 11 for c results in:
$39a + 65c + 2cd = 39a + 715 + 22d$
With just a and d remaining, it is clear that we want a to be bigger since it is multiplied by 39 while d is only multiplied by 22. Substituting 7 for a and 5 for d results in:
$39a + 715 + 22d = 273 + 715 + 110 = 1098$.
If all are substituted at once:
$3ab + 5bc + 2cd = 3(7)(13) + 5(13)(11) + 2(11)(5) = 1098$.

7. 1098

8. First label the side of the square as x, the radius of the large circle as R, and the radius of the small circle as r. AF is equal to $R\sqrt{2}$ since it is the diagonal of a square with side length R. So FB which is equal to x since it is the radius of the quarter circle is also equal to $R + R\sqrt{2}$. FD is the diagonal of the square, so its length is $x\sqrt{2} = (R + R\sqrt{2})\sqrt{2} = R\sqrt{2} + 2R$.
$BD = FD - FB = R\sqrt{2} + 2R - (R + R\sqrt{2}) = R$.
In the same way that $FA = R\sqrt{2}$, $CD = r\sqrt{2}$.
So $BD = r + r\sqrt{2}$, but we know that $BD = R$.
Therefore, $R = r + r\sqrt{2} = r(1 + \sqrt{2})$ meaning that R is $(1 + \sqrt{2})$ times as long as r which is what the question asked for.

8. $\sqrt{2} + 1$



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1. If A is the amount Allison made, then Suzi made $1.4A$. If Suzi gets a 20% raise she would make $1.2(1.4A) = 1.68A$. Allison getting a 30% raise means she now makes $1.3A$. For Allison to be making as much as Suzi, she would need to raise her pay by $.38A$ from the $1.3A$ she is currently making. That is a $\frac{.38A}{1.3A} = \frac{38}{130} \approx .2923 \approx 29\%$ increase in pay.

1. 29

2. There are $3! = 6$ orders the three 6th graders can stand in, there are $4! = 24$ orders the four 7th graders can stand in, and there are $5! = 120$ orders in which the five 8th graders can stand. If they are arranged 6th graders, then 7th graders, then 8th graders, there would be $6 \cdot 24 \cdot 120 = 17{,}280$ possible orders. However, there are $3! = 6$ orders in which the three grade levels can be arranged. That's $17{,}280 \cdot 6 = 103{,}680$ possible orders.

2. 103680

3. In a regular octagon, each interior angle has $180 - \frac{360}{8} = 135°$. $m\angle ABJ = 135 - m\angle CBJ = 135 - 56.34 = 78.66°$. Angles ABJ and IJB have a sum of $180°$ because they are on the same side of a transversal of two parallel lines. Thus, $m\angle IJB = 180 - 78.66 = 101.34°$. $\angle IJK$ is the interior angle of a regular hexagon which makes its measure $120°$.
So $\angle BJK = 360 - 120 - 101.34 = 138.66°$.

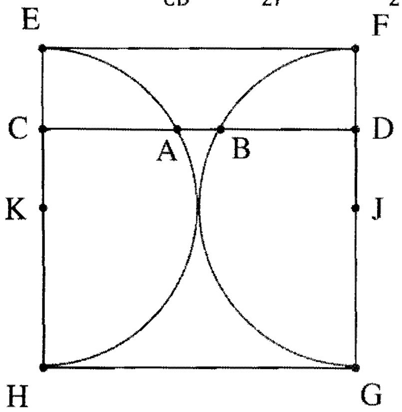
3. 138.66

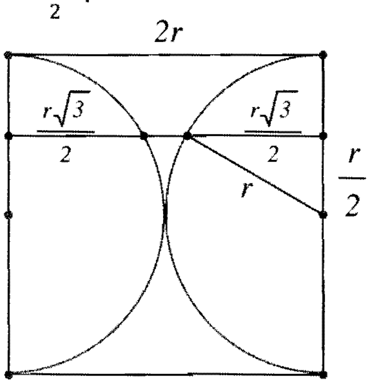
4. Listing the quad-primes we find the following pairs: $(3,7), (7,11), (13,17), (19,23), (37,41), (43,47), (67,71), (79,83), (97,101), (103,107), (109,113)$. Thus, there are a total of 11 pairs with both primes less than 120.

4. 11

5. Calling the radius of the circle r makes the sides of the square equal to $2r$. Drawing the radius from J to B creates a right triangle that with the Pythagorean Theorem allows finding $\text{BD} = \frac{r\sqrt{3}}{2} = \text{AC}$. That makes $\text{AB} = 2r - 2\left(\frac{r\sqrt{3}}{2}\right) = 2r - r\sqrt{3}$. That makes $\frac{\text{AB}}{\text{CD}} = \frac{2r - r\sqrt{3}}{2r} = \frac{r(2-\sqrt{3})}{2r} = \frac{2-\sqrt{3}}{2}$.

5. $\frac{2-\sqrt{3}}{2}$





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6. Converting all the numbers, including the exponents, from base four to base ten results in the new expression shown below.
$2^{11} + 3^{10} + 2^{23} + 11^{10} \rightarrow 2^5 + 3^4 + 2^{11} + 5^4$
$2^5 + 3^4 + 2^{11} + 5^4 = 32 + 81 + 2048 + 625 = 2786$
To convert to base 4, break 2786 into powers of 4.
$2(1024) + 2(256) + 3(64) + 2(16) + 0(4) + 2(1) = 2786$
$2(4^5) + 2(4^4) + 3(4^3) + 2(4^2) + 0(4^1) + 2(4^0) = 2786$
So converting 2786 to a base 4 number results in 223202_4

6. 223202

7. If the diameter of the circle is d, then the height of the rectangle is d and the width is $3d$. Using the Pythagorean Theorem with the height and width as the legs and the diagonal of the rectangle as the hypotenuse we have:
$d^2 + (3d)^2 = 20^2$
$d^2 + 9d^2 = 400$
$10d^2 = 400$
$d^2 = 40$
The area of the rectangle is $d \cdot 3d = 3d^2 = 3(40) = 120$ sq in.

7. 120

8. To find how many zeros $m!$ factorial ends in you need to divide n by 5 and round any decimal down. That quotient is how many numbers in that factorial product are divisible by 5. If the quotient is greater than or equal to 5, then there are some numbers in the factorial product that are divisible by 25 and have 2 factors of 5 in them. Dividing the quotient by 5 and rounding down indicates how many numbers are divisible by 25. As long as the quotient is greater than or equal to 5, continue the process to find factors of 125, 625 or greater powers of 5. Factorials from 0! to 4! end in 0 zeros, 5! to 9! end in 1 zero, from 10! to 14! end in 2 zeros, 15! to 19! end in 3 zeros, 20! to 24! end in 4 zeros, and 25! to 29! end in 6 zeros. So it is not possible for $m!$ to end in 5 zeros. Following this pattern, we find that the next number of zeros $m!$ can't end in is 11. Every 6^{th} whole number will not be possible to make, but when m hits 125, an extra factor of 5 gets introduced and an additional value of m is impossible. This happens for every five values that aren't possible. The list of values for m that aren't possible are: 5, 11, 17, 23, 29, 30, 36, 42, 48. That's 9 values of m less than or equal to 50 that are not possible.

8. 19

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9. $$\left(\frac{\left(\frac{2}{3}\right)^{-4}\times\left(17\frac{1}{2}\right)^{-3}\times\left(\frac{4}{9}\right)^{5}}{\left(\frac{3}{7}\right)^{3}\times\left(2\frac{1}{2}\right)^{-3}\times\left(\frac{3}{8}\right)^{-2}}\right)^{-7} =$$

9. 52

$$\left(\frac{\left(\frac{3}{2}\right)^{4}\times\left(\frac{2}{35}\right)^{3}\times\left(\frac{2}{3}\right)^{10}}{\left(\frac{3}{7}\right)^{3}\times\left(\frac{2}{5}\right)^{3}\times\left(\frac{8}{3}\right)^{2}}\right)^{-7} =$$

$$\left(\frac{\frac{2^{13}\times 3^{4}}{2^{4}\times 3^{10}\times 5^{3}\times 7^{3}}}{\frac{2^{9}\times 3^{3}}{3^{2}\times 5^{3}\times 7^{3}}}\right)^{-7} =$$

$$\left(\frac{\frac{2^{9}}{3^{6}\times 5^{3}\times 7^{3}}}{\frac{2^{9}\times 3^{1}}{5^{3}\times 7^{3}}}\right)^{-7} =$$

$$\left(\frac{2^{9}\times 5^{3}\times 7^{3}}{2^{9}\times 3^{7}\times 5^{3}\times 7^{3}}\right)^{-7} = \left(\frac{1}{3^{7}}\right)^{-7} = \left(\frac{3^{7}}{1}\right)^{7} = 3^{49} = x^{y}$$

Thus, $x + y = 3 + 49 = 52$.

10. The x-intercepts of $y = 6x^2 - x - 15$ are found by replacing y with 0 and solving for x.

10. $\frac{551}{144}$

$0 = 6x^2 - x - 15$
$0 = (3x - 5)(2x + 3)$
$0 = 3x - 5$ or $0 = 2x + 3$
$x = \frac{5}{3}$ or $x = -\frac{3}{2}$

The y-intercepts of $x = 12y^2 - 13y - 14$ are found by replacing x with 0 and solving for y.
$0 = 12y^2 - 13y - 14$
$0 = (3y + 2)(4y - 7)$
$0 = 3y + 2$ or $0 = 4y - 7$
$y = -\frac{2}{3}$ or $y = \frac{7}{4}$

Since the vertices of the quadrilateral are on the axes, the diagonals of the polygon will be the axes and are perpendicular. To find the area of a polygon with perpendicular diagonals, use the formula $A = \frac{d_1 \cdot d_2}{2}$.

The horizontal diagonal equals $\frac{5}{3} + \frac{3}{2} = \frac{10}{6} + \frac{9}{6} = \frac{19}{6}$.

The vertical diagonal equals $\frac{7}{4} + \frac{2}{3} = \frac{21}{12} + \frac{8}{12} = \frac{29}{12}$.

$A = \left(\frac{19}{6} \cdot \frac{29}{12}\right) \div 2 = \frac{551}{72} \cdot \frac{1}{2} = \frac{551}{144}$

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