1. $(-13) - (8 - 31) - (-7) =$
$(-13) - (-23) - (-7) =$
$(-13) + 23 + 7 = 17$

1. 17

2. If Mike rolls a three 20% of the time, that means he rolls a three $0.2(40) = 8$ times. Likewise Sean would roll a three $0.25(40) = 10$ times. So Sean rolled a three $10 - 8 = 2$ more times than Mike.

2. 2

3. By definition, whole numbers begin with 0 and increase by 1. There are 11 different addition problems that use two whole numbers with a sum of 10 as listed below:

$(0 + 10), (1 + 9), (2 + 8), (3 + 7), (4 + 6), (5 + 5), (6 + 4),$
$(7 + 3), (8 + 2), (9 + 1)$ and $(10 + 0)$

3. 11

4. If Arjun's age now is A, then his age in 10 years would be $A + 10$. His age 10 years ago would have been $A - 10$. The equation $A + 10 = 5(A - 10)$ represents the situation given in the problem. Solving the equation will give us Arjun's age.
$A + 10 = 5(A - 10)$
$A + 10 = 5A - 50$
$60 = 4A$
$15 = A$

4. 15

5. If we call the largest of the eight numbers x, we can write the equation :
$(x - 7) + (x - 6) + (x - 5) + (x - 4) + (x - 3) + (x - 2) + (x - 1) + x = 68$
Solving the equation will give us the largest number.
$8x - 28 = 68$
$8x = 96$
$x = 12$

5. 12

6. Since it doesn't matter what order Callie chooses her friends in this problem is called a *Combination*. To choose 4 out of 10 things the following formula should be used:
${}_{10}C_4 = \frac{10!}{4!(10-4)!} = \frac{10!}{4!\cdot 6!} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210.$

6. 210

7. Taking the numbers from the stem and leaf plot and listing them results in 23, 24, 24, 31, 35, 37, 39, 52, 56, 58, 60, 67. Because there are twelve numbers in the list, the median number would be the average of the 6th and 7th numbers in the list. In this case, that is the average of 37 and 39, which is 38.

7. 38

8. From 100 to 199, there are 19 numbers that have at least one 7 as one of their digits. (There are ten that have a 7 in the tens place 70 - 79 and ten that have a 7 in the units place 107, 117, ...197. That would be 20 numbers, except that 177 would be counted twice so only 19.) There will also be 19 numbers in the 200s, 300s, 400s, 500s, 600s, 800s and 900s, while all hundred numbers in the 700s will have a 7 in them. That is a total of $19 \times 8 + 100 = 252$.

8. 252

9. A square with area 256 in² would have side length $\sqrt{256} = 16$ and have a perimeter of $16 \times 4 = 64$ in. A square with area 169 in² would have side length $\sqrt{169} = 13$ and have a perimeter of $13 \times 4 = 52$ in. The new square has a perimeter that is $64 - 52 = 12$ in shorter.

9. 12

10. Call the two numbers x and y. We can write two equations to fit the given information. $x + y = 30$ and $x - y = 4$. We are looking for $x^2 - y^2$. That expression is known as the difference of two squares and can be factored as follows:
$x^2 - y^2 = (x + y)(x - y)$.
Since we know both $x + y = 30$ and $x - y = 4$ we can substitute those values into the expression as follows:
$x^2 - y^2 = (x + y)(x - y) = (30)(4) = 120$.

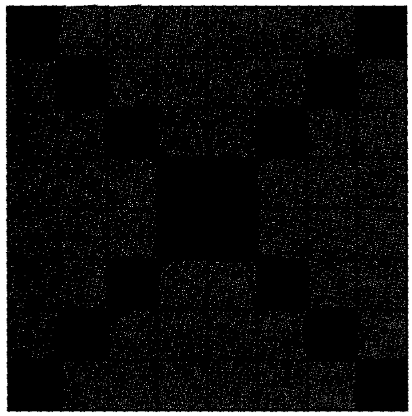
10. 120

11. If 10 pencils cost \$1.20 and the price is reduced 20%, the new price becomes 1.20(.8) = 0.96. (We multiplied by (.8) because taking 20% off the price means you have to pay for 80% of the price and 80% = 0.8.) The discounted price of 10 pencils is 0.96, so the discounted price of 20 pencils would be 2(.96) = \$1.92.

11. 1.92

12. There would be 8 black squares along each diagonal with no double counting of the squares. So of the 64 total squares, 16 would be black. $\frac{16}{64} = \frac{1}{4} = 25\%$

12. 25



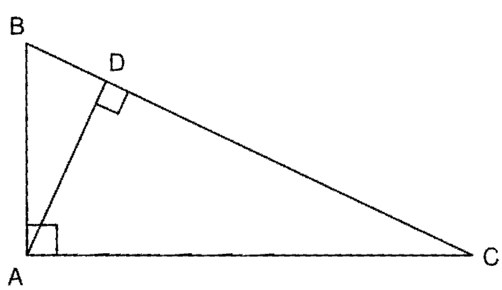
13. The first child has 5 choices for the flavor they take. The next next has 4 choices, the 3rd child has 3 choices, the 4th child has 2 choices and the last child has just the 1 choice (whatever is left). That gives $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways to distribute the popsicles to the 5 children.

13. 120

14. First use the Pythagorean Theorem to find that AC = 24. $AB^2 + AC^2 = BC^2 \rightarrow 100 + AC^2 = 676 \rightarrow AC^2 = 576 \rightarrow$ AC = 24. Since the area of a triangle is base times height divided by two, there are now two base/height pairs that can be used to find the area. If the base is 24 then the height is 10. If the base is 26 then the height must be AD. Whichever pair is used, the value must be the same since the area isn't going to change.

$AD \times 26 = 24 \times 10 \rightarrow AD = \frac{240}{26} = \frac{120}{13} = 9\frac{3}{13}$.

14. $9\frac{3}{13}$



Sally is going to save 630 - 588 = 42 cents if she uses SureCopy according to the chart below.

15. 42

			CopyPro		SureCopy
	12 black		72		96
24 2-sided					
	12 color		180		240
	18 black		108		72
36 1-sided					
	18 color		270		180
		Totals	630		588

The original rectangle had length l and width w. The new rectangle would have length $l - 2$ and width $w + 5$. The original area would have been lw, while the new area becomes $(w + 5)(l - 2)$. The new area is 60 sq. ft. larger so the following equation applies: $(w + 5)(l - 2) = lw + 60$. That simplifies to: $-2w + 5l = 70$. A second equation describing the original perimeter is $2w + 2l = 70$. By adding those two equations together the result is $7l = 140$ and $l = 20$ which also yields $w = 15$ by plugging 20 into l in the perimeter equation. If the original dimensions were 15 by 20, the new dimensions are 20 by 18 with an area of 360 square feet.

16. 360

For this problem you can either change both numbers to base 10 and then find their product, or multiply the two numbers in base 5 and convert the product to base 10. Using the first method, $22_5 = 2(5) + 2 = 12$ and $34_5 = 3(5) + 4 = 19$. Since $12 \times 19 = 228$, the units digit that we are looking for is 8. By the second method :

17. 8

$$\begin{array}{r} 22_5 \\ \times\ 34_5 \\ \hline 143_5 \\ 1210_5 \\ \hline 1403_5 \end{array} = 228$$

18. In an arithmetic sequence each term increases or decreases by the same amount. This sequence is getting bigger, so we know the common difference between terms is positive. The increase from the 2^{nd} term to the 7^{th} term is $53 - (-22) = 75$. If the sequence increases 75 over 5 terms then the common difference is $\frac{75}{5} = 15$. To go from the 7^{th} term to the 31^{st} term is a total of 24 terms further along in the sequence that increases 15 each time. That means that the 31^{st} term must be $53 + 24(15) = 53 + 360 = 413$.

18. 413

19. The reciprocals of natural numbers are all going to be of the from $\frac{1}{n}$. A fraction in simplest form will repeat if the denominator is divisible by a prime number other than 2 or 5. Since a fraction is more likely to repeat than terminate (come to an end) it will be easier to count the possible denominators that are divisible by only 2 and 5 and therefore terminate, and then subtract that from 100 to find the ones that repeat. By making a careful list you can find the following natural numbers that only have 2 and 5 as prime factors. A nice way to start is to list the powers of 2 and then the powers of 5 that are less than 100: 1, 2, 4, 8, 16, 32, 64 and 1, 5, 25. Then multiply each of the numbers in the 5 list by each in the power of 2 list stopping when you reach 100 and being sure not to repeat anything you already have: 10, 20, 40, 80, 50, 100. Combining these lists we get one total list: 1, 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80, 100. That's 15 numbers whose reciprocals terminate, so the other $100 - 15 = 85$ numbers must have reciprocals that repeat.

19. 85

20. If there are 13 Jumpers, and only $\frac{1}{6}$ of Skippers become Jumpers, then there must have been a total of $13 \times 6 = 78$ Skippers. If those 78 Skippers represent $\frac{3}{8}$ of all Hoppers, then we could use the proportion $\frac{3}{8} = \frac{78}{x}$ to find the number of Hoppers. From the proportion we get:
$3x = 8(78) \rightarrow 3x = 624 \rightarrow x = 208$

20. 208

1. The smallest circle has radius $\frac{1}{2}$ the largest circle, so the area of the smallest circle is $\left(\frac{1}{2}r\right)^2 \pi = \frac{1}{4}r^2\pi$ or $\frac{1}{4}$ the area of the large circle. That makes the area of the smallest circle equal to 16. The middle circle has a radius that is $\frac{3}{4}$ the radius of the largest circle, so its area is $\left(\frac{3}{4}r\right)^2 \pi = \frac{9}{16}r^2\pi$ or $\frac{9}{16}$ the area of the largest circle which would be $\frac{9}{16}(64) = 36$. So the area of the outer ring is 64 – 36 = 28, the area of the middle ring is 36 – 16 = 20, and the middle circle has an area of 16. In the outer ring, 5 of the 20 sections are shaded, so $\frac{1}{4}$ of the ring is shaded. In the middle ring only 3 of the 20 sections are shaded, so $\frac{3}{20}$ is shaded. In the center circle 5 of 20 sections are again shaded, so $\frac{1}{4}$ is shaded. We can now find the total shaded area as:

$\frac{1}{4}(28) + \frac{3}{20}(20) + \frac{1}{4}(16) = 7 + 3 + 4 = 14.$

21. 14

2. To simplify this problem it would help to eliminate the If we multiply the ratio by 12, we will get a ratio using only whole numbers. Making $\frac{1}{2}$ to $\frac{1}{3}$ to $\frac{1}{4}$ become 6 to 4 to 3. Since it is a ratio, multiplying them all by the same number does not change how much they get at all. You can see that Sam's $\frac{1}{2}$ was originally twice as much as Xyla's $\frac{1}{4}$ and now his 6 is double her 3. The ratio has not changed. It is important to note that it does not say Sam gets half the money!! So if we work with the ratio 6:4:3, we can say there are 6 + 4 + 3 = 13 parts and Sam gets 6 of them. Dividing the \$52,000 into 13 parts makes each part equal to \$4,000. If Sam gets 6 of those that is \$4,000 × 6 = \$24,000. To be safe it would also mean that Rami gets \$4,000 × 4 = \$16,000 and Xyla get \$4,000 × 3 = \$12,000 for a total of: \$24,000 + \$16,000 + \$12,000 = \$52,000 which is the correct total. You could also have just added the fractions to get $\frac{13}{12}$ and noticed that Sam gets $\frac{6}{12}$ from the $\frac{13}{12}$ or $\frac{6}{13}$ of the total.

22. 24,000 or 24,000.00

23. To find out how many seconds it will take Floaty we need to first divide 1.46×10^7 by 7.3 to get a quotient of 2,000,000. Then we need to divide by 60 to find the number of minutes and divide by 60 again to get the number of hours. Finally we need to divide by 12 since Floaty flies 12 hours per day. This gives us: $\frac{2000000}{60\times60\times12} = \frac{20000}{6\times6\times12} = \frac{5000}{3\times3\times12} = \frac{1250}{3\times3\times3} = \frac{1250}{27} \approx$ 46.2. Since you can't use a calculator on the Sprint Round, this method of simplifying before doing the division can save you a lot of time and trouble.

23. 46

24. $3x^2 + 17x - 28 = x^2 + 6x - 7 \rightarrow 2x^2 + 11x - 21 = 0$

Once you have simplified the equation to get zero on the right, you can use the quadratic formula to find the solutions of x. For equations of the form $ax^2 + bx + c = 0$ the quadratic formula $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ will help find the solutions of x.

$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x = \frac{-11\pm\sqrt{11^2-4(2)(-21)}}{2(2)} = \frac{-11\pm\sqrt{289}}{4} = \frac{-11\pm17}{4} = \frac{6}{4}$ or $\frac{-28}{4}$

The sum of those two solutions would be $\frac{6}{4} + \frac{-28}{4} = \frac{-22}{4} = \frac{-11}{2}$. The polynomial $2x^2 + 11x - 21 = 0$ could also be factored to $(2x - 3)(x + 7) = 0$. This means that either $2x - 3 = 0$ or $x + 7 = 0$ and therefore $x = \frac{3}{2}$ or $x = -7$. It is important to note that this sum can always be found without actually knowing either or the solutions. The sum of the solutions of a quadratic equation is always equal to $\frac{-b}{a}$ which in this case is $\frac{-11}{2}$.

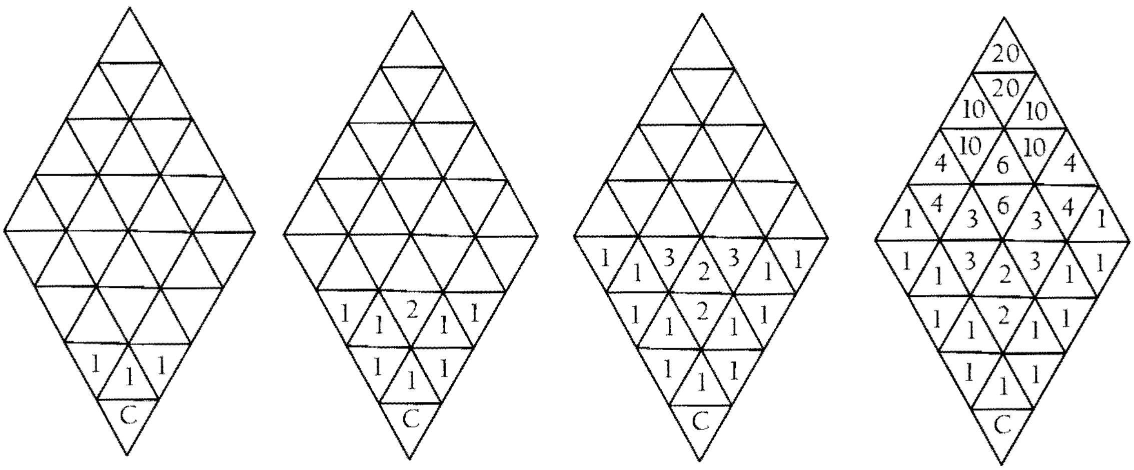
24. $\frac{-11}{2}$

25. We first start by noting that 8 beads can be lined up in 8! ways. However, since they are on a circular string (necklace) they can be rotated to look as if any one of the eight beads was chosen first, so we need to divide that total by 8. Finally, because the necklace could be flipped upside down without creating a new necklace, but reversing the order in which the beads appear, we need to further divide by 2. Therefore the total number of ways the beads can be arranged is:
$8! \div 8 \div 2 = 2520$.

25. 2520

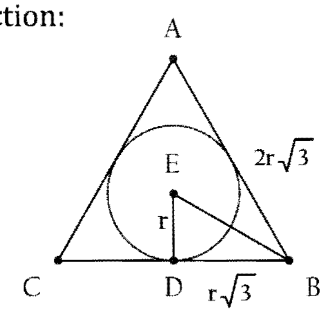
. A nice way to find the number of ways to go from the top to the bottom is to actually start at the bottom. Filling in how many ways you can get to C from the very bottom is easy as shown in the first diagram. Moving up one line at a time by looking at the options from each triangle can help fill in the number of ways to C from each triangle. In the second diagram, there is a triangle marked with a 2. This is because from that triangle you could go down/right or down/left. Each of those choices brings you to a triangle with 1 path, and $1 + 1 = 2$. Continue that process as shown in the last two diagrams. We find 20 paths from the top, M, to C.

26. 20



27. $\frac{\pi\sqrt{3}}{9}$

If we call the radius of the circle r and recognize that the small triangle that has been drawn in is half of an equilateral triangle, then we can find the other sides of the small triangle in terms of r. Side $\overline{DB} = r\sqrt{3}$ which makes a side of the large triangle equal to $2r\sqrt{3}$. The area of the circle would be πr^2 while the area of an equilateral triangle can be found with the formula $A = \frac{s^2\sqrt{3}}{4} = \frac{(2r\sqrt{3})^2\sqrt{3}}{4} = \frac{(4r^2\cdot 3)\sqrt{3}}{4} = \frac{12\sqrt{3}r^2}{4} = 3\sqrt{3}r^2$.
The fraction of the area of the triangle that is inside the circle would then be represented by the fraction:
$\frac{\pi r^2}{3\sqrt{3}r^2} = \frac{\pi}{3\sqrt{3}} = \frac{\pi\cdot\sqrt{3}}{3\sqrt{3}\cdot\sqrt{3}} = \frac{\pi\sqrt{3}}{9}$.



28. Since we don't know how far the distance is from Hakeem's home to his work we could either define a variable, or choose a distance that makes the 60 mph and the 45 mph easier to work with. The LCM of 60 and 45 is 180, so let's use that. If the distance to his work is 180 miles, it would have taken him 3 hours to get there at 60 mph. The return trip at 45 mph would have taken him 4 hours. In total, he would have traveled 360 miles in 7 hours. Dividing those two tells us the average speed would be $\frac{360}{7} = 51\frac{3}{7}$ mph. There is actually a formula to produce that same answer, but the explanation is very lengthy. Try looking at this: $\frac{2}{\frac{1}{60}+\frac{1}{45}} = 51\frac{3}{7}$.

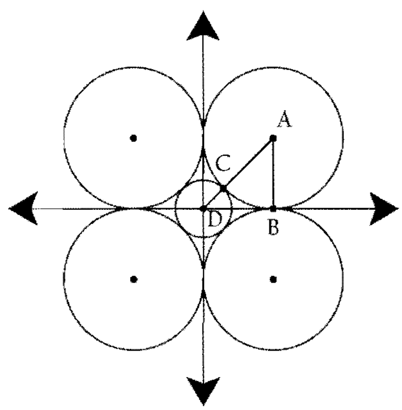
28. $51\frac{3}{7}$

29. If Billy gets voted off in the third week that means he did not get voted off in either the 1st or 2nd weeks. The probability of not getting voted off the 1st week is $\frac{9}{10}$, the probability of not getting voted off the 2nd week is $\frac{8}{9}$, and the probability of getting voted off the 3rd week is $\frac{1}{8}$. For all three of those things to happen, we have to calculate the product of those probabilities: $\frac{9}{10} \cdot \frac{8}{9} \cdot \frac{1}{8} = \frac{1}{10}$. Alternatively, each contestant is equally likely to get voted off on the third week, so Billy's likelihood is $\frac{1}{10}$.

29. $\frac{1}{10}$

30. In the diagram shown, AB = 1 and DB = 1. By the Pythagorean Theorem we can find that AD = $\sqrt{2}$. We know that AC = 1 since it is the radius of the circle, so the radius of the smaller circle is $\sqrt{2} - 1$. The area of the smaller circle will be $(\sqrt{2} - 1)^2\pi = (2 - 2\sqrt{2} + 1)\pi = (3 - 2\sqrt{2})\pi = A\pi$ so A must be equal to $3 - 2\sqrt{2}$.

30. $3 - 2\sqrt{2}$



1. Each scoop is a sphere and the formula for the volume of a sphere can be found with the formula $V = \frac{4}{3}\pi r^3$. Since the radius of each scoop is 3 and there are three scoops the total volume would be $3\left(\frac{4}{3}\pi 3^3\right) = 4\pi \cdot 27 = 108\pi$. The volume of a cylinder is found by multiplying the area of the circular base (B) times the height. We can find the area of the base of the cylinder ($\pi r^2 = \pi(3.5)^2 = 12.25\pi$). Since we are looking for the height, we can call that H, and use the equation $V = BH \rightarrow 108\pi = 12.25\pi H$. Dividing both sides by 12.25π tells us that $H = \frac{108\pi}{12.25\pi} = \frac{108}{12.25} \approx 8.8$.

1. 8.8

2. The first step will be to find the prime factorization of 84 million. The prime factorization of 84,000,000 = $2^8 \cdot 3^1 \cdot 5^6 \cdot 7^1$. Any number that is a factor of 84 million must have a prime factorization that uses the same prime factors raised to powers that do not exceed those from the prime factorization of 84 million. So a factor could have 2 as a prime factor with an exponent from 0 to 8, but because we want an *even* factor, the number must be divisible by 2. Meaning that the exponent on 2 cannot be 0. That gives us 8 choices (1 through 8) for the exponent. A factor of 84 million could have a 3 as a prime factor with 2 choices for the exponent (0 or 1). A factor of 84 million could have a 5 as a prime factor with 7 choices for the exponent (0 through 6). A factor of 84 million could have a 7 as a prime factor with 2 choices for the exponent (0 or 1). In total then, there are $8 \cdot 2 \cdot 7 \cdot 2 = 224$ numbers that would be an even factor of 84 million.

2. 224

3. From 100 to 199, there are 18 numbers that have exactly one 8 as one of their digits. (There are ten that have a 8 in the tens place 80 - 89 and ten that have a 8 in the units place 108, 118, ...198. That would be 20 numbers, except that 188 would be counted twice when we don't want to count it at all since it doesn't have *exactly* one 8, so only 18 since we counted a number twice that we didn't want to count at all.) There will also be 18 numbers in the 200s, 300s, 400s, 500s, 600s, 700s, and 900s. In the 800s all the numbers have at least one 8, but 19 of the numbers have an additional 8, so only 81 that have *exactly* one 8. That is a total of: $18 \times 8 + 81 = 225$.

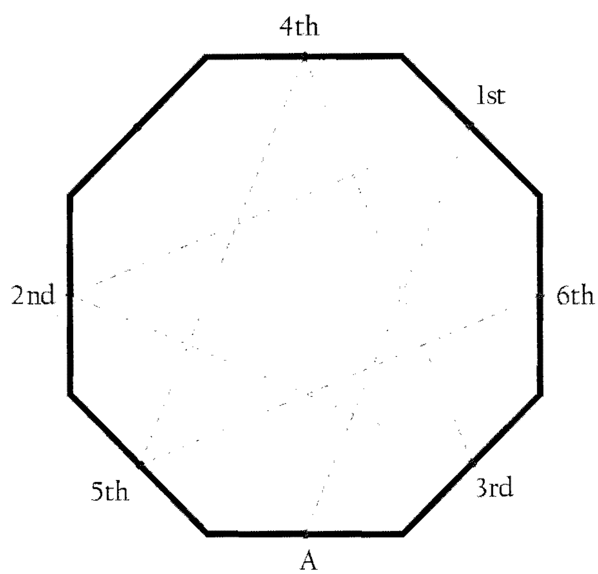
3. 225

4. When a cube is painted and cut up into unit cubes, the 8 corners are always painted on 3 faces. The unit cubes on each of the 12 edges (not including the corner cubes) of the cube are painted on 2 faces. On the face of the large cube, the unit cubes that do not lie on the edges of the large cube are only painted on 1 face. Finally, the unit cubes that do not have any of their faces on the exterior of the large cube are not painted at all. Therefore, we only want to count the unit cubes that lie along the edges of the largest cube. If there are n unit cubes along each edge, but we do not want to count the corner cubes at each end, then there are $n - 2$ unit cubes on each edge that are painted on 2 faces. Since there are 12 edges on a cube that means there are a total of $12(n - 2) = 12n - 24$ unit cubes painted on 2 sides.

4. $12n - 24$

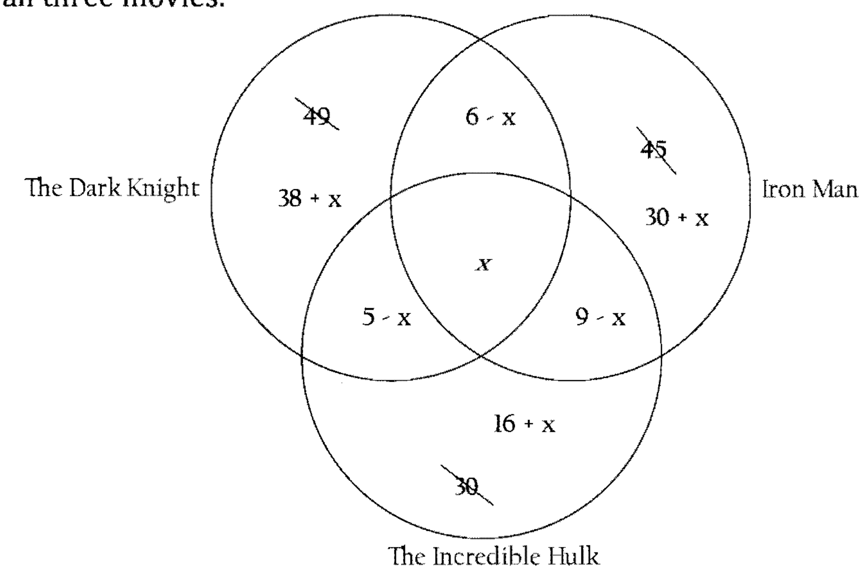
5. The diagram below shows that the laser would hit the X on its 6th reflection. If the laser loses 10% each time it hits a mirror that means 90% of it continues on each time. If the laser hits a mirror 6 times, then 90% of 90% of 90% of 90% of 90% of 90% continues on. So we need to calculate $(90\%)^6$ to find out what percent continues on after hitting the 6th mirror. $(.9)^6 \approx .531 = 53.1$ %.

5. 53.1



6. To solve a problem like this we will want to use a Venn Diagram like the one below. Start by placing the number of people that saw each movie in the appropriate circle. Then in the overlap spaces we place the number of people that saw 2 of the movies. Doing this, however, means we are saying more people saw each movie. For example placing the 6 and 5 in the overlaps of *The Dark Knight* means that the diagram says 49 + 5 + 6 = 60 people saw *The Dark Knight.* To correct this we subtract 11 from the original 49 we placed in *TheDark Knight* circle. Then repeat those same steps for the other two circles. Now because we are looking for the number that saw all 3, we can put an x in the center space. However, that means there are now 6 + x that saw *The Dark Knight* and *Iron Man.* In order to correct that, we must change the 6 in the overlap to a 6 – x. We need to repeat that in each overlap spot. However, that means that now each circle's total is x less than it was supposed to total. We fix that by adding x into each of the original circles. Now add up everything in all the circles to get a sum of 104 + x. We want the total to be 110 – 4 = 106. So solving 104 + x = 106 to get x = 2, meaning that 2 people must have seen all three movies.

6. 2



7. The hands of a clock will start in the same place at 12 o'clock. The next time they are in the same place will be around 1:06. The next time they meet will be around 2:11. There will be a meeting of the hands every hour until 10:55. There will not be a meeting during the 11 o'clock hour as the hands next meet at 12 o'clock again. That means that they meet eleven times every twelve hours, or 22 times a day. In a week that's 22 × 7 = 154 times.

7. 154

8. There are three colors that could go in any of the 4 rectangles. Because there are three choices for each of the four rectangles there are a total of $3 \cdot 3 \cdot 3 \cdot 3 = 81$ ways to paint the rectangles. However, because they can't all be the same, we need to subtract the three cases in which all 4 rectangles are painted the same color. That gives us $81 - 3 = 78$ ways to paint the rectangles under the given conditions.

8. 78

1. If the side of the original square is s, then the area of the square is $s^2 = 60$ sq. cm. To increase the side lengths of the square by a certain percentage it is easiest to multiply the side lengths by a decimal that is greater than 1 by the decimal equivalent of the percent (for example to raise something 40%, multiply it by 1.4). If we call that amount x, the new side of the square would be xs and the area would be $(xs)^2 = x^2s^2 = 101.4$. Replacing s^2 by 60 results in the equation $60x^2 = 101.4$. Solving results in $x^2 = 1.69$ and $x = 1.3$. So the sides of the square have been multiplied by 1.3 which means they have been increased by 30%.

1. 30

An alternate approach would be to find the difference in side lengths and divide by the original side length as represented by the computation below. The answer of 1.3 again means that the new square's sides are 1.3 times as big as the original square's sides. Be careful not to do any rounding during the problem, though, as that would result in an approximation of the correct answer.

$$\frac{\sqrt{101.4}-\sqrt{60}}{\sqrt{60}} = 1.3$$

2. Using the variable n to represent the number of nickels, there would be $2n$ dimes and $2n - 2$ quarters. Multiplying the number of each type of coin by their respective values in cents and finding the sum will give the total value in cents as the equations below show. Using d as the number of dimes and q as the number of quarters, we get the following equations $d = 2n$ and $q = 2n - 2$.
Using substitution we can arrange the first equation so that the variable n is used.

$5(n) + 10(d) + 25(q) = 250$
$5(n) + 10(2n) + 25(2n - 2) = 250$
$5n + 20n + 50n - 50 = 250$
$75n - 50 = 250$
$75n = 300$
$n = 4$

Thus, there are 4 nickels, 8 dimes and 6 quarters for a total of 18 coins.

2. 18

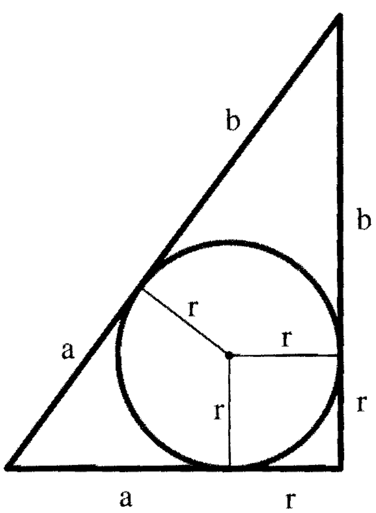
It would be possible to rewrite the original equation so that only the variable d is used by substituting $\frac{d}{2}$ for n and $d - 2$ for q. Solving for d would result in $d = 8$. Thus, there are 4 nickels, 8 dimes and 6 quarters as found in the first method.

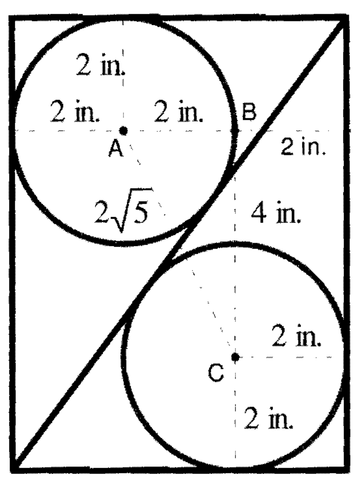
3. If the interior angle of a polygon is a whole number, then the exterior angle would also be a whole number. The sum of the exterior angles is always 360 degrees, so if the interior and exterior angles are whole numbers then the number of angles must be a factor of 360. The total number of sides in all the polygons would then be the sum of the factors of 360 (excluding 1 and 2 since there are no 1 or 2 sided polygons). The sum of the factors of 360 can be found without listing them by the following steps:
$360 = 2^3 \cdot 3^2 \cdot 5^1$
The sum of the factors of 360 is:
$(2^0 + 2^1 + 2^2 + 2^3)(3^0 + 3^1 + 3^2)(5^0 + 5^1)=$
$(1 + 2 + 4 + 8)(1 + 3 + 9)(1 + 5) =$
$(15)(13)(6)=1170$
The total number of sides in the set of polygons is:
$1170 - (1 + 2) = 1167$

3. 1167

4. The first thing needed is the radius of the circle. By drawing the radii of the circle to each of the points where the circle is tangent to the triangle, the sides of the triangle are each divided into parts than can be labeled r, a, and b as shown in the diagram on the left. With legs of 6 and 8, the hypotenuse of the triangle is 10. The three equations $a + r = 6$, $r + b = 8$ and $a + b = 10$ lead to finding that $r = 2$. With that known, the diagram on the right can be labeled as it is. Since points B and C are each 2 inches from the right side of the rectangle, the line BC must be parallel to the right side, meaning that BC must be vertical. We also know that AB is horizontal, which means we can form right triangle ABC with the hypotenuse AC being the distance between the two centers. The legs of triangle ABC are 2 inches and 4 inches, so using the Pythagorean Theorem we can find the hypotenuse to be $\sqrt{40} = 2\sqrt{5}$ inches.

4. $2\sqrt{5}$





$3^6 + 9^3 + 3^{13} + 27^6$
$3^6 + 3^6 + 3 \cdot 3^{12} + 3^{18}$
$2 \cdot 3^6 + 3 \cdot (3^6)^2 + (3^6)^3$
Replacing 3^6 with x gives the expression:
$2x + 3x^2 + x^3$
$x(2 + 3x + x^2) = x(x^2 + 3x + 2)$
$x(x + 1)(x + 2)$
That is the product of three consecutive integers.
$x + 1 = 3^6 + 1 = 730$ is the middle integer.

5. 730

If ◈+♁+♁=19, then ◈+◈+♁+♁+♁+♁=38. We also know that ♁+♁+♁+◈+◈=32, which means that ♁=6. By plugging 6 into the equation, we can find that ◈=7. By plugging 6 and 7 into the first equation in the problem we find that ◎=8, which then leads to finding that §=5. Thus, the final expression then has a value of:
◎+◎+◈+◈+◈+♁+§+§=8+8+7+7+7+6+5+5=53.

6. 53

Any point on the perpendicular bisector of a segment is equidistant from the endpoints of that segment. If there exists a point that is always equidistant from any 2 of the 3 points, then the perpendicular bisectors of the three segments formed by the 3 points must all intersect at one point. The segment AB has a slope of $m = \frac{5-(-1)}{10-4} = \frac{6}{6} = 1$. The midpoint of the segment is $\left(\frac{10+4}{2}, \frac{5+(-1)}{2}\right) = (7, 2)$. The equation of the perpendicular bisector would have a slope of -1 (the opposite of the reciprocal of the segment's slope) and pass through the point $(7, 2)$. The equation would be $(y - 2) = -1(x - 7)$ which simplifies to $y = -1x + 9$. The segment AC is a horizontal segment with a midpoint of $(3, 5)$. The perpendicular bisector then would be a vertical line passing through $(3, 5)$ and have the equation $x = 3$. Those two perpendicular bisectors intersect somewhere on the line $x = 3$, so the x coordinate must be 3. Plugging in 3 to the equation $y = -1x + 9$ gives $y = -1(3) + 9 = -3 + 9 = 6$. That means they intersect at the point $(3, 6)$. You could find the equation of the third perpendicular bisector and see where it intersects the other two bisectors, but the only place they could all possibly intersect is the only point where the two we already checked intersect.

7. (3 , 6)

8. There are $9! = 362{,}880$ orders in which to arrange the 9 digits. In some of those orders though the digits 1, 2 and 3 are not in order from least to greatest. Since there are $3! = 6$ ways to arrange the 1, 2 and 3, only 1 out of every 6 of the 362,880 orders have the 1, 2 and 3 in order. So there are $362{,}880 \div 6 = 60{,}480$ ways to arrange the 9 digits with the 1, 2 and 3 in the order we desire.

8. 60480

9. To be composite, a number must be the product of two natural numbers greater than 1. Since we want the 7 smallest that also aren't divisible by 2, 3, 4, 5, 6, 8, 9 or 10, the smallest factors the numbers could be divisible by are 7, 11, 13, 17, 19... The smallest composite numbers divisible by 7 but not any of the other numbers are 49, 77, 91, 119, 133, etc. The smallest composite numbers divisible by 11 are 121, 143, 187, etc. Together, the smallest 7 are 49, 77, 91, 119, 121, 133 and 143. The sum is 733.

9. 733

10. If the code is three single-digit primes then there are 4 choices for the first digit, 3 choices for the second (since it can't be the same as the first), and 3 choices for the last digit (since it can't be the same as the second). That's $4 \cdot 3 \cdot 3 = 36$ possible codes. If the code is a single-digit prime followed by a 2-digit prime, there are 4 choices for the single-digit prime and 20 choices for the 2-digit prime (13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97... don't count 11 since it has two digits in a row that are the same). That's $4 \cdot 20 = 80$ codes. But of those 80, some will break the rule of 2 digits in a row being the same, and some will have already been counted in the first 36 codes. The ones with 2 digits in a row would be 2|23, 2|29, 3|31, 3|37, 5|53, 5|59, 7|71, 7|73 and 7|79 (that's 9 that shouldn't be counted). The one's where all the digits are prime and were counted in the original 36 would be 2|37, 2|53, 2|73, 3|23, 3|53, 3|73, 5|23, 5|37, 5|73, 7|23, 7|37 and 7|53 (that's 12 that were double counted). So there are a total of $36 + 80 - 9 - 12 = 95$ codes that fit the rules.

10. 95