1. $5 + 2(10 - 7)^2 =$
$5 + 2(3)^2 =$
$5 + 2(9) =$
$5 + 18$
23

1. 23

2. The first 30% off brings the price down to 70% of the original. The next 30% comes off of the 70% that is left, so it is actually 30% of 70% which is 0.3(70%) =21% more off. Taking 21% off the remaining 70% leads to 70% - 21% = 49%, so now you are left paying for 49%. If you are paying for 49% of the original price, then that means that 51% has been taken off the price!!

2. 51

3. If Jonathan has 4 errands to run he has 4 choices for the first errand, 3 choices for the second errand, 2 choices for the third errand, and 1 choice for which to do last. That gives Jonathan $4 \times 3 \times 2 \times 1 = 24$ orders in which to run the 4 errands!!

3. 24

4 $\frac{x+3}{x-4} = \frac{5}{4}$
Solve this proportion by cross multiplying.
$4(x + 3) = 5(x - 4)$
$4x + 12 = 5x - 20$
$12 = x - 20$
$32 = x$

4. 32

5. If there are 18 with buttons and 15 that are blue that would be a total of 33 shirts. But there were only 27 shirts, so where did the extra 6 come from? The extra 6 are the ones that were counted twice!!!!

5. 6

6. If he needs his over-all average to be 91, the sum of the 4 quizzes must be $91 \times 4 = 364$. So far, the sum of his first three quizzes is $95 + 82 + 89 = 266$. So on the last test he must get the rest of the $364 - 266 = 98$ points he needs.

6. 98

7. You can solve this with an equation by making x the starting floor $x + 6 - 3 + 10 - 4 + 5 - 7 + 2 - 13 = 1 \rightarrow x - 4 = 1 \rightarrow x = 5^{th}$ floor

7. 5^{th}

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8. If the units digit and tens digit are the same, the sum of the digits would be double the units digit. Thus, we are looking for integers whose tens digit is larger than the units digit (or integers whose units digit is less than the tens digit). Since we know the integers must be greater than 50, start with a tens digit of 5 and list the units digits that could work.
5: 4, 3, 2, 1 → 4 integers (not 0 since integer > 50)
6: 5, 4, 3, 2, 1, 0 → 6 integers
7: 6, 5, 4, 3, 2, 1, 0 → 7 integers
8: 7, 6, 5, 4, 3, 2, 1, 0 → 8 integers
9: 8, 7, 6, 5, 4, 3, 2, 1, 0 → 9 integers
For a total of $9 + 8 + 7 + 6 + 4 = 34$ integers!!

8. 34

9. $\frac{17}{35}$ $\frac{21}{43}$ $\frac{12}{25}$ $\frac{36}{73}$ $\frac{28}{57}$
All of these fractions are very close to $\frac{1}{2}$ but just a little bit smaller, so the question is which is closest to $\frac{1}{2}$ since the one closest to $\frac{1}{2}$ will be the largest. The amount that each is actually less than $\frac{1}{2}$ is .5 over the denominator. Whichever of these new fractions is smallest will correspond to the one closest to $\frac{1}{2}$. For instance, $\frac{17}{35}$ is $\frac{.5}{35}$ less than $\frac{1}{2}$. Since these new fractions all have the same numerator, the new fraction with the biggest denominator makes that missing piece the smallest and the original fraction the biggest (closest to a half). That means that $\frac{36}{73}$ is the largest.

9. $\frac{36}{73}$

10. Based on the problem we can set up equations using b to represent the number of bicycles and t to represent the number of tricycles. Since both types of cycles use only 1 seat, we can say $b + t = 28$. Additionally, since we know bicycles have 2 wheels each and tricycles have 3 wheels each we can say $2b + 3t = 67$. If we double the first equation we'd get $2b + 2t = 56$. Comparing that to the second equation we can see that the extra t must equal 11. A second, less algebraic solution, is so say that if every seat is used to make a bicycle, there would be $28 \times 2 = 56$ wheels used. The extra $67 - 56 = 11$ wheels would be used to turn 11 bicycles into tricycles.

10. 11

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11. $\frac{2x-y}{x+y} = \frac{2}{3}$ 11. $\frac{5}{4}$

Solve by cross multiplying.
$3(2x - y) = 2(x + y)$
$6x - 3y = 2x + 2y$
$4x = 5y$
$\frac{x}{y} = \frac{5}{4}$

12. $10x + y$ represents any two-digit number where x is the tens digit and y is the units digit. Reversing the digits gives us $10y + x$. We can make the equation: 12. 17

$10x + y + 9 = 10y + x$
$9x + 9 = 9y$
$x + 1 = y$

So the units digit must be 1 more than the tens digit: 12, 23, 34, 45, 56, 67, 78, 89 are the 8 numbers that work that are less than 100. Between 100 and 200 there are also these 9 numbers 101, 112, 123, 134, 145, 156, 167, 178, 189 for a total of 17(be careful not to miss that we can now use 101, where we couldn't use 01 before since it is not considered a two-digit number).

13. 13. $\frac{60}{43}$

$$\frac{2}{1+\frac{1}{2+\frac{1}{3+\frac{1}{4}}}} = \frac{2}{1+\frac{1}{2+\frac{1}{\frac{13}{4}}}} = \frac{2}{1+\frac{1}{2+\frac{4}{13}}} = \frac{2}{1+\frac{1}{\frac{30}{13}}} = \frac{2}{1+\frac{13}{30}} = \frac{2}{\frac{43}{30}} = \frac{60}{43}$$

14. Using substitution, we can take $2x + 3y + 4z$ and replace the y with $2x$ and the z with $3y$ to get: 14. 32
$2x + 3(2x) + 4(3y) = kx$
Now replace y again with $2x$ to get:
$2x + 6x + 4(3(2x)) = kx$
$2x + 6x + 24x = kx = 32x$
$k = 32$

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15. Since he has the same number of each type of coin we should find the value of a set of coins containing exactly 1 penny, 1 nickel, 1 dime, 1 quarter and 1 half dollar. We find that each set of coins containing 1 of each type of coin has a value of $1 + 5 + 10 + 25 + 50 = 91$ cents. Now we can divide 546 by 91 to see that he must have $546 \div 91 = 6$ sets of coins that each contain 1 of each type of coin. Thus he has $6 \times 5 = 30$ coins.

15. 30

16. Slope $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{z-5}{28-4} = \frac{z-5}{24} = \frac{2}{3}$

Using cross multiplication:

$3(z - 5) = 2(24)$

$3z - 15 = 48$

$3z = 63$

$z = 21$

16. 21

17. If the two numbers are x and y, the sum of their reciprocals is: $\frac{1}{x} + \frac{1}{y} = \frac{y}{xy} + \frac{x}{xy} = \frac{x+y}{xy}$. From the problem we know the sum $(x + y)$ is 13, and the product (xy) is 35 so $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{13}{35}$.

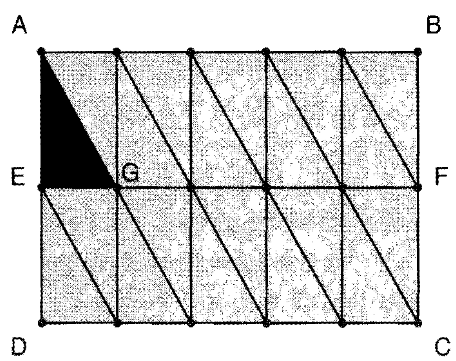
17. $\frac{13}{35}$

18. The sum of the interior angles of a convex polygon is always a multiple of 180 degrees. The closest multiple of 180 to 1578 is 1620. So the missing angle would have to be 1620 - 1578 = 42 degrees. If we look at the next larger sum of interior angles (1800°) we would find the remaining angle to have a measure of 1800 - 1578 = 222 degrees, but in a convex polygon all angles are less than 180.

18. 42

19. By dividing up the original diagram into equal pieces we create the diagram below. Remember, we want the ratio of the triangle to the area NOT in the triangle. Since the triangle is 1 piece and the rest is 19 pieces, the ratio is $\frac{1}{19}$.

19. $\frac{1}{19}$



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20. Interior angle of a pentagon = $180 - \frac{360}{5} = 180 - 72 = 108°$. Interior angle of an octagon = $180 - \frac{360}{8} = 180 - 45 = 135°$. Thus the ratio of the measure of the interior angle of a regular pentagon to the interior angle of a regular octagon is $\frac{108}{135} = \frac{36}{45} = \frac{4}{5}$.

20. $\frac{4}{5}$

21. Walking clockwise from the two to the nine is $\frac{7}{12}$ of the the circle's circumference. The circumference of the circle is equal to π times the diameter. If the radius is 18 ft, then the diameter is 36π ft. So we are looking for $\frac{7}{12}$ of 36π which is 21π.

21. 21π

22. If Darwin is splitting his jelly beans up over several days, he is dividing the total by how much he eats each day. First we need a common unit, so we should change 20 kilograms into milligrams. 20 kilograms = 20,000 grams = 20,000,000 milligrams. Now we can determine the number of days by dividing: 20,000,000 ÷ 8 = 2,500,000 days. Now change 2,500,000 to scientific notation which is 2.5×10^6.

22. 2.5×10^6

23. Let's call the smallest number x. Since the angles form an arithmetic sequence, each angle increases by the same amount. Call that amount y. So the 4 angle measures are $x, x + y, x + 2y$, and $x + 3y$ degrees. Since those four angles must add up to 360 degrees, $4x + 6y = 360$ which simplifies to $2x + 3y = 180$. We want to find the biggest of the 4 numbers, so we want the first one(x) to be as small as possible. Start plugging in 1, 2, 3 ... for x to find the first whole number that works for y (y must be a whole number since the second smallest number, $x + y$, is a whole number.) You'll find that when $x = 3, y = 58$. So the biggest of the 4 numbers can be found two ways. Either $3 + 3(58) = 177$ or just 180 - 3 = 177°.

23. 177

24. If you start multiplying 1,234,567 by itself you will see pretty quickly that you only need to multiply the units digits to find the units digit of the answer. If we look at powers of 7 (the units digits) we see that it forms this pattern: 7, 9, 3, 1, 7, 9, 3, 1... That shows that the units digit repeats every 4 powers and ends in 1 every 4th. So the 88th power would end in 1 and the 89th power would end in $1 \times 7 = 7$.

24. 7

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25. There are seven periods to work with. He has 7 choices for the red block, 6 for the blue block, 5 for the green block and 4 for the yellow block. That is a total of $7 \times 6 \times 5 \times 4 =$ 840 ways to make the schedule. Only one would match what is currently used at Clarke Middle School, so the probability is $\frac{1}{840}$.

25. $\frac{1}{840}$

26. When the sides of any polygon double, the area of that object becomes 4 times as big. So the new area would be $20 \times 4 =$ 80 yd^2. Since there are 3 ft in 1 yd, there are (3^2) ft^2 in (1^2) yd^2, or 9 square feet per square yd. That means there are $80 \times 9 = 720$ square feet of area in which the turtle can play.

26. 720

27. Writing an equation we get:

$4\left(\frac{1}{2\pi r}\right) = 2r$

$\frac{4}{2\pi r} = 2r$

$4 = 4\pi r^2$

$1 = \pi r^2 =$ Area of the circle

27. 1

28. The left side of the equations $xy - 8x - 3y = 0$ can almost be factored but there is a part missing. If we try to factor we get $(x - 3)(y - 8)$. If we multiply that back out though we'd get $xy - 8x - 3y + 24$. We find ourselves with an extra $+ 24$. Well luckily this is an equation so we could add 24 to both sides of the original!!! That gives us $xy - 8x - 3y + 24 = 24$ which is equal to $(x - 3)(y - 8) = 24$. So the product of two numbers is 24. We want the largest possible value of either variable, and the largest factor of 24 is 24, so one possibility would be: $(x - 3)(y - 8) = 1 \times 24$. So $x - 3 = 1$ and $y - 8 = 24$. That would make $y = 32$, the biggest we could make it!!!

28. 32

29. Since half of what is left when Hao drinks will still be lemonade and $\frac{3}{8}$ of all 6 cups is still lemonade we can write the equation: $\frac{1}{2}(6 - x) = \frac{3}{8}(6)$, where x is the amount Hao drank, and $6 - x$ is the amount of liquid left after he drank. (Recall that $\frac{3}{8} = 37.5\%$.) Solve and you get $3 - \frac{x}{2} = 2\frac{1}{4} \rightarrow \frac{3}{4} = \frac{x}{2} \rightarrow x = 1.5$ cups.

29. 1.5

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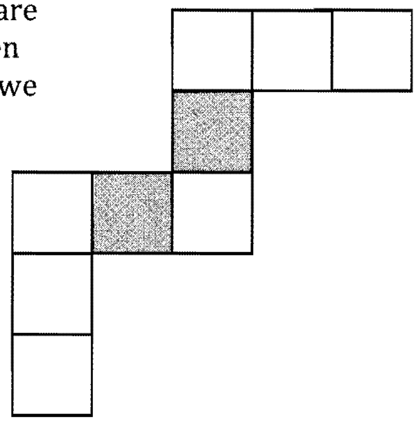
30. Since there are 4 rows or columns that have to have the same sum, we can call that sum x, and we would say the sum of the four rows/columns is $4x$. If we want x to be as big as possible we would want $4x$ to be as big as possible. We can do that by putting the biggest numbers in the spots that are counted twice. The squares where a column meets a row are all counted twice and there are three of them. They happen to be the three squares adjacent to the shaded squares. If we put the 9, 8 and 7 in the spots that count twice we would be adding:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 9 + 8 + 7 = 69 = 4x$.

However, 69 is not divisible by 4, but 68 is. So we put 9, 8 and 6 in the spots that count twice for a total of 68 and each row/column is $68 \div 4 = 17$. Now if we place the 8 and 9 in the same row/column they would already equal 17, so we must put the 6 between them. Now let's put the 8 above the upper right shaded square and the 9 to the left of the lower left shaded square (or reversed – either way will give the same result). One row will have 6 and 9 for a sum of 15, thus the letter between them must be 2 to make a total of 17. The other will have 6 and 8 for a sum of 14, thus the letter between them must be 3 to make a total of 17. So A and B must be 2 and 3, creating a sum of 5.



30. 5

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1. In a six hour period, the hour hand will travel 6 hr or half way around the clock while the minute hand will make 6 full trips around the circle. The circumference of the circle made by the tip of the hour hand is $2\pi(6\text{in}) = 12\pi$ inches and half of that would be 6π. The circumference of the circle formed by the tip of the 7 hand is $2\pi(8\text{ in}) = 16\pi$ inches and 6 trips around would be 96π. That is $96\pi - 6\pi = 90\pi \approx$ 283 inches more that the hour hand.

1. 283

2. Think about this... pick any four digits and arrange them in order from greatest to least. There is only one way to do that right? Well that is what is happening in this problem. There are 10 digits to choose from and you need to choose 4 of them. It doesn't matter what order you pick them in since you will put them in order yourself. So all we need to know is how many ways can you pick 4 out of 10 things. That is called a Combination and looks like this: ${}_{10}C_4$ and is calculated like this:

$${}_{10}C_4 = \frac{10!}{(10-4)!\cdot 4!} = \frac{10!}{6!\cdot 4!} = \frac{10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1\cdot 4\cdot 3\cdot 2\cdot 1} = \frac{10\cdot 9\cdot 8\cdot 7}{4\cdot 3\cdot 2\cdot 1} = 210$$

2. 210

3. Since there is no information about the actual number of kids we could make it up. If there are 100 girls then there must be 300 boys. Since 60% of 100 = 60 and 40% of 300 = 120 that gives us 180 out of the total 400 kids who scored above a 30.
$\frac{180}{400} = \frac{18}{40} = \frac{9}{20} = \frac{45}{100} = 45\%$.

Another way to do this is with Algebra. If we call the number of girls g, and the number of boys $3g$, we would have: $.6g + .4(3g) = .6g + 1.2g = 1.8g$ kids who scored above a 30. The total number of kids is $g + 3g = 4g$. Thus,
$\frac{1.8g}{4g} = \frac{1.8}{4} = \frac{18}{40} = 45\%$

Yet another way to do this is to say that three-fourths of the kids are boys, so the average must be three fourths of the way from the girls score to the boys score. Since the scores are 20% apart, and three-fourths of 20% is 15%, the average must be 15% from the girls towards the boys which is: 60% – 15% = 45%.

3. 45

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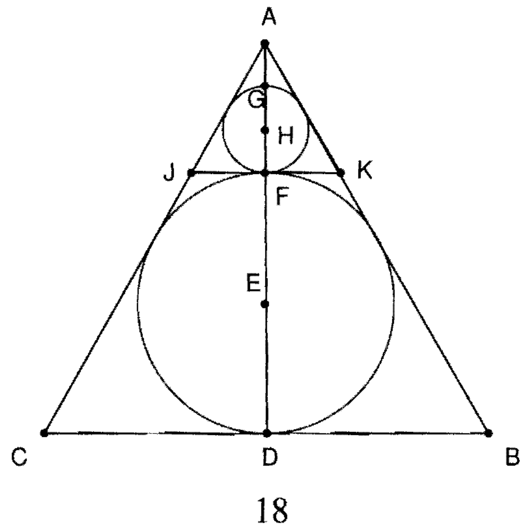
4. In the diagram below I added a line from A to D through key points G, H, F, and E in that order. Since this is an equilateral triangle, AD cuts CB in half, thus DB = 9 units. You can use the Pythagorean Theorem, or you might just know that AD = DB times $\sqrt{3}$. So AD = $9\sqrt{3}$ units. When a circle is inscribed in an equilateral triangle the diameter of the circle is two-thirds of the height. So FD = $6\sqrt{3}$ units. That makes AF = $3\sqrt{3}$ units. We now have a new equilateral triangle AKJ. The smaller circle inside AJK again has a diameter two-thirds of the height of the triangle. Two-thirds of $3\sqrt{3}$ is $2\sqrt{3}$ and the radius is half of that and is $\sqrt{3}$. So the area of the circle equals $\pi r^2 = \pi(\sqrt{3})^2 = 3\pi$ sq. units.

4. 3π



5. Andrew counts like this: 7, 12, 6, 3, 8, 4, 2, 1, 6, 3, ...
Hweedo counts like this: 17, 14, 7, 4, 2, 1, −2, −1, −4, −2, ...

5. 16

The sum of Andrew's first 10 numbers is 52.
The sum of Hweedo's first 10 numbers is 36.

The difference is 52 – 36 = 16.

6. Write an equation where the original population is p. Also, note that taking 11% off is the same as finding the 89% that is left.

6. 10,000

$.89(p + 1200) = p - 32$
$.89p + 1068 = p - 32$
$1100 = .11p$
$10{,}000 = p$

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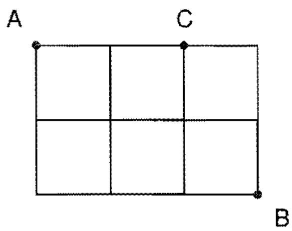
7. To get from point A to point B, Katie has to take 5 steps. Each step must be either right or down. You have to choose which 2 of the 5 steps will be down. That is a combination again ${}_5C_2 = \frac{5\cdot4}{2\cdot1} = \frac{20}{2} = 10$. In order to go through C, Katie would have to go straight across the first 2 steps. From there Katie needs to take 3 steps and you need to choose one of those to be to the right. That is also a combination, but picking one of 3 is easy, there's just 3 ways. So that gives us 3 out of 10 going through C.

7. $\frac{3}{10}$



8. The first thing we need to determine is what the biggest possible denominator that can be made so that each piece would be as small as possible. If the two fractions have denominators of 8 and 9, then their common denominator would be 72. The biggest fraction I could make then would be $\frac{71}{72}$. Now we need to see if it is possible. So we want to add the fractions $\frac{x}{8} + \frac{y}{9}$ which would give us: $\frac{9x+8y}{72}$. This means we need to add a multiple of 8 to a multiple of 9 to get a sum of 71. So list the multiples of each:

8. $\frac{71}{72}$

8: 8, 16, 24, 32, 40, 48, 56, 64, ...
9: 9, 18, 27, 36, 45, 54, 63, 72, ...

Now can we find a pair that adds up to 71? YES!
$63 + 8 = 71$
$9(7) + 8(1) = 9x + 8y$
So $x = 7$ and $y = 1$
Giving us the fractions $\frac{7}{8} + \frac{1}{9} = \frac{71}{72}$.

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1. The 10 students that averaged 80% scored a total of $10 \times 80 = 800$ points. The second groups had 20 kids who scored a total of $20 \times 85 = 1700$ points. If the third group made up 50% of the students, the other 50% must have been the 30 kids already accounted for. So the third group of 30 kids scored a total of $30 \times 90 = 2700$ points. That's a total of 60 kids and a total of 5200 points. That's an average of $5200 \div 60 = 86.\bar{6} \approx 87\%$, to the nearest whole number.

1. 87

2. Triangle ADF is a right triangle with right angle at D. The hypotenuse AF is the length of two squares, while the short leg of the triangle is the length of one square. That means that the triangle is a 30-60-90 triangle (half of an equilateral triangle). Angle AFD = 30°, so angle FJE = 60° as they are the two acute angles of a right triangle and are complementary. Angle DJE is supplementary to angle FJE so DJE = 120°.

2. 120

3. To convert through several different units, we should start by creating rates for the trade values in the problem.
$\frac{9C}{7H}, \frac{5H}{12P}, \frac{15J}{14P}, \frac{xC}{16J}$
Any of those rates can be inverted, and multiplying the rates can create new rates.
$16J \times \frac{14P}{15J} \times \frac{5H}{12P} \times \frac{9C}{7H} = 8C$
So for 16 Junkuwunkas you could get 8 Chumpawumpas.

3. 8

4. If it would take Stan 18 hours, he could complete $\frac{1}{18}$ of the wall per hour. Likewise Stan could complete $\frac{1}{15}$ and Luanne would complete $\frac{1}{24}$ of the wall per hour. Together they would complete $\frac{1}{18} + \frac{1}{15} + \frac{1}{24} = \frac{20}{360} + \frac{24}{350} + \frac{15}{360} = \frac{59}{360}$ of the wall per hour. If t is the time it takes them to finish the wall, the equation $\frac{59}{360}t = 1$ can find how long it will take to finish. Multiplying both sides by $\frac{360}{59}$ results in $t = \frac{360}{59}$. That means it would take them $\frac{360}{59} = 6\frac{6}{59}$ hours to finish the wall together.

4. $6\frac{6}{59}$

5. Since each word must have a vowel, each word will get an "o" and since the two "o's" are indistinguishable from each other there is only one way to split up the vowels. To form the two-letter word you have 3 choices for the consonant, and then 2 choices for which letter would come first. Once the consonant is chosen for the two letter word the rest will just go to the three-letter word, but those three letters can be arranged in $3 \times 2 \times 1 = 6$ ways. That means there are a total of $3 \times 2 \times 6 = 36$ ways to form the two "words".

5. 36

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The way that Martina passes out the money doesn't actually matter since we are not looking for the probability but just the number of ways the money could be split up. For 20 dollars to be split up between three people we can think about placing 2 dividers between the 20 bills. The easiest way to do that is to add two more bills to the 20 for a total of 22. Line those 22 up and then choose two of the bills to actually be the dividers. The other 20 will now be divided into 3 groups representing the 3 grandchildren. So there are ${}_{22}C_2=\frac{22!}{(22-2)!\cdot 2!}=\frac{22\cdot 21}{2\cdot 1}=231$ ways.

6. 231

First we need to notice that because FC is the diameter of the circle which makes angles FEC and FDC are both right angles. We can use the Pythagorean Theorem on right triangle GDC to find the length of GC. $CD^2+GD^2=CG^2 \rightarrow$ $6^2+2^2=CG^2 \rightarrow 36+4=CG^2=40 \rightarrow CG=\sqrt{40}=2\sqrt{10}$
Also, in right triangle FDC we know FC=10 and CD = 6 which by the Pythagorean Theorem FD=8 (Pythagorean Triple 6-8-10). Because GD =2, FG = 8 - 2 =6. Power of a Point tells us that when two chords (FD and CE) intersect in a circle the products of the parts they are broken into will be equal. In this case CG×GE=FG×GD .

7. $\frac{9\sqrt{10}}{5}$

$2\sqrt{10}\times GE=6\times 2$

$GE=\frac{12}{2\sqrt{10}}\times\frac{\sqrt{10}}{\sqrt{10}}=\frac{12\sqrt{10}}{20}=\frac{3\sqrt{10}}{5}$

Now use the Pythagorean Theorem with GE and FG to find FE.

$FE^2+GE^2=FG^2$

$FE^2+\left(\frac{3\sqrt{10}}{5}\right)^2=6^2$

$FE^2+\frac{90}{25}=36$

$FE^2=32\frac{2}{5}=\frac{162}{5}$

$FE=\sqrt{\frac{162}{5}}=\frac{9\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{9\sqrt{10}}{5}$

Billy runs a total of 150 yd at 20 ft/sec. That is 450 ft at a rate of 20 ft per sec. It will take Billy $450 \div 20 = 22.5$ sec. To find how far Mike ran we need to use the Pythagorean Theorem. $150^2+300^2=\text{Mike}^2$(distance converted to feet).

8. 14.9

$22{,}500+90{,}000=\text{Mike}^2$

$112{,}500=\text{Mike}^2$

$335.41\approx$ Mike

If Mike needs to run 335.41 ft in 22.5 sec (arriving the same time as Billy) he will need to run at a rate of $\frac{335.41}{22.5}\approx 14.9$ ft/sec

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9. The chart at the right shows that based on symbols that Sean and Juan could show, Sean would win twice, Juan would win once, and they would tie once out of every four matches. So Sean wins $\frac{2}{4}$ of each match. But since there is a tie $\frac{1}{4}$ of the time a second round must be held $\frac{1}{4}$ of the time to determine who would win the first round. During that second round though $\frac{1}{4}$ of the matches would end in a tie again, and a 3rd round would be needed to determine who would win the 1st match. Since there is always a $\frac{1}{4}$ chance of a tie, it is possible it could take forever to determine who won just the 1st match. Just because it *could* go on forever doesn't mean the probability can't be determined. The probability that Sean would win the 1st match would be the sum of the infinite geometric sequence:

$$\frac{2}{4}+\frac{1}{4}\left(\frac{2}{4}\right)+\left(\frac{1}{4}\right)^2\left(\frac{2}{4}\right)+\left(\frac{1}{4}\right)^3\left(\frac{2}{4}\right)+\cdots=\frac{\frac{2}{4}}{1-\frac{1}{4}}=\frac{\frac{2}{4}}{\frac{3}{4}}=\frac{2}{3}.$$

So Sean would win any round $\frac{2}{3}$ of the time and Juan would win $\frac{1}{3}$ of the time. This could also be determined from the diagram as Sean wins 2 of the 3 outcomes that do not result in a tie and therefore a redo.

For Sean to be the first to win 2 matches, the contest would have to go one of these ways where S is Sean winning and J is Juan winning: SS, JSS, SJS which would be calculated as:

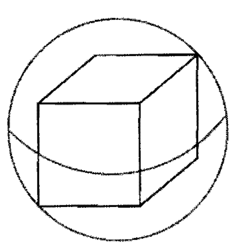
$$\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)+\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)+\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)=\frac{4}{9}+\frac{4}{27}+\frac{4}{27}=\frac{20}{27}.$$

9. $\frac{20}{27}$

Juan \ Sean	Paper	Scissors
Paper	TIE	SEAN
Rock	SEAN	JUAN

10. If the side length of a cube is 12, then the space diagonal of the cube is $12\sqrt{3}$. The diagonal of the cube is the diameter of the circumscribed sphere. So the radius of the sphere is $6\sqrt{3}$, and the surface area of the sphere is $4\pi(6\sqrt{3})^2 = 432\pi$. The surface area of the cube is $6 \times 12^2 = 6 \times 144 = 864$. That means the surface area of the sphere is $432\pi - 864$ greater than the surface area of the cube.

10. $432\pi - 864$



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